REFRACTION THROUGH A PRISM-REVISITED

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Abstract

The minimum deviation method using large angle prisms is not an effective method for refractive index determination. An accurate method of determining refractive index is discussed in this paper.

Introduction

In optics a prism is a transparent body which has two inclined refracting surfaces. The angle between its two refracting surfaces is called the refracting angle or angle of the prism (marked A in Figure-1). The optical properties exhibited by a prism are:

1. Dispersion
2. Total internal reflection and
3. Deviation.

Based on the above properties some constant values of an optically transparent media are obtained using it in the form of a prism. Figure-1 shows the path of a ray in an equilateral prism with BA and AC as its refracting surfaces. BC is an unpolished surface and is called the base of the prism. N1 and N2 are the normals at the surfaces 1 and surface 2 respectively. With reference to Figure-1, i_1 = angle of incidence, i_2 = angle of emergence, d = deviation, r_1= angle of refraction at first surface, and r_2 =angle of refraction of the second surface.

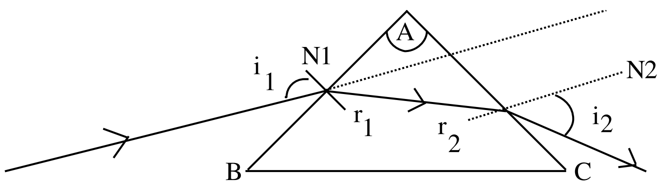

Figure 1: Refraction through a prism

Standard text books, like Jenkins and White [1], give the relation between the above parameters as:

$d = i_1 + i_2 - A$ …1
$A = r_1 + r_2$ …2

A plot connecting i_1 with i and d is called (i – d) curve. From the graph d_{min} = D is obtained and the refractive index of the prism is obtained using

$$n = \frac{\sin i}{\sin r}$$

At D, $i_1 = i_2$; $r_1 = r_2$ and using (1) and (2)

$$n = \frac{\sin(\frac{A+D}{2})}{\sin\frac{A}{2}} \quad \ldots 3$$

The (i-d) curve

The usual method of determining the refractive index is to draw the (i – d) curve and to obtain the minimum deviation from the graph and calculated using equation-3. The theoretical (i – d) curve is obtained by substituting the value of i_2 in equation $n = \sin i_2/\sin r_2$. After some rearrangements we get the equation (1) as for $i_1 = i$,

$$d = i + \sin^{-1}[n \sin (A-\sin^{-1}\frac{\sin i}{n})] - A \quad \ldots 4$$

for $i_1 = i$,

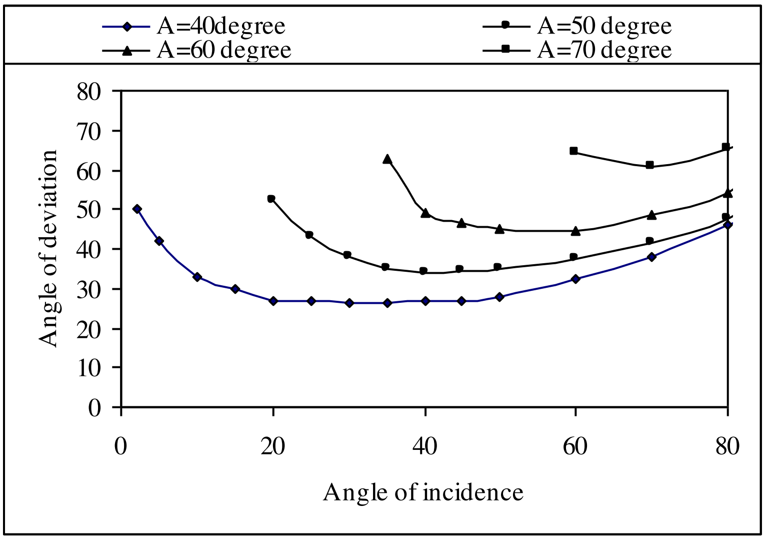

Figure 2: The i-d curve for A = 40°, 50°, 60, and 70

A detailed study of the (i – d) curve for different prisms with different angles was done by Udayanandan [4] based on equation-4 to demonstrate the convenience of using a prism of angle $A=60^0$ in classroom experiments. Even though theoretically D can be calculated accurately, the determination of D is not accurately possible experimentally for an observer by visual observation alone. This is clear from the (i – d) curve for prisms of angle $A=70^0,60^0,50^0,40^0$ for n =1.6. A sharp minimum deviation is not obtained experimentally from the graph as a result of which a small variation in D results in large errors in estimation of *n*.

Minimum angle of incidence

Another drawback of the minimum deviation method is the lack of information about the minimum angle of incidence. Freedman [6] had already pointed out that there is a minimum angle of incidence for a given prism. If the angle of incidence is less than this angle the refracted ray will be totally internally reflected. There are different methods for obtaining an expression for i_{min}. of which a method is given here.

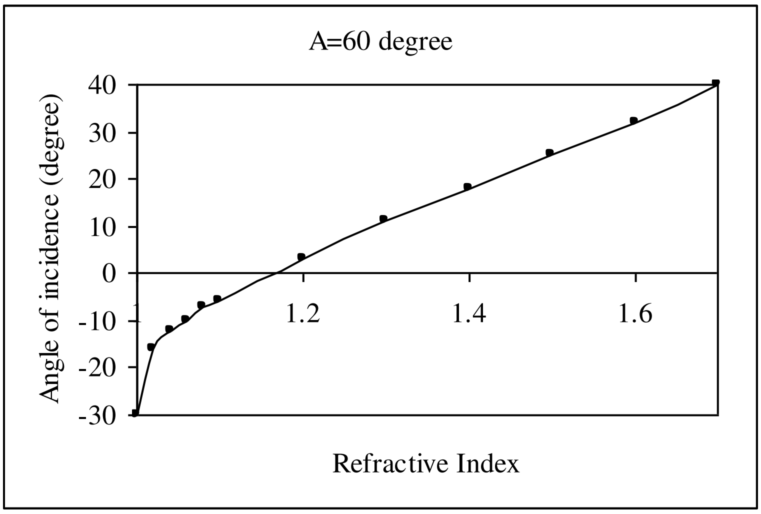

Figure 3: The (i_{min} - n) curve for A=60^0

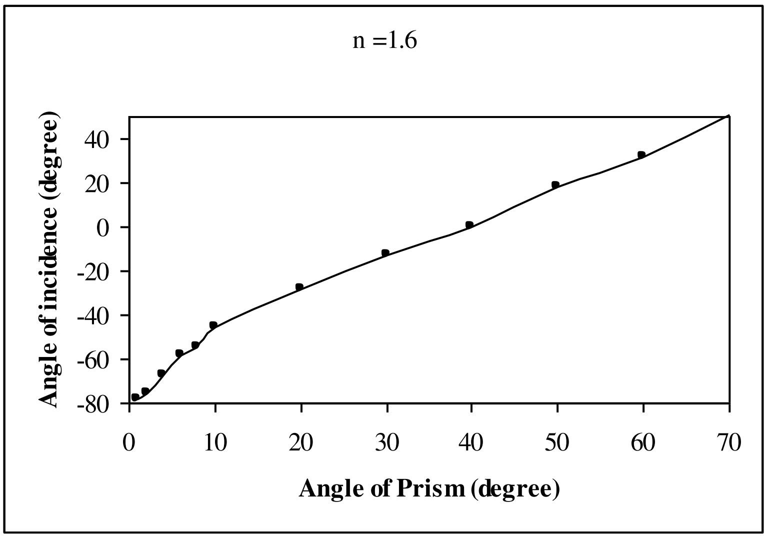

Figure 4: The i_{min} -A curve for n=1.6

When $i_1 = i = i_{min}$, $r_1 = r_{min}$ and $r2 = r_{max} = n \sin(A - C)$, where $C = r_{max} = \sin^{-1}(1/n)$
This simplifies to

$$i_{min} = \sin^{-1}(\sqrt{n^2 - 1}\sin A - \cos A) \qquad \ldots 5$$

A graph between i_{min} and n is given in Figure-3, for $A=60^0$ prism.

The graph will help in having apriori knowledge of the minimum angle of incidence. For example if the prism is of angle 60^0 and *n* is about 1.6, then a choice of angle of incidence less than 30^0 is not advisable. Another graph between minimum angle of incidence and A is given which highlights the importance of negative angle of incidence for small angle prisms.

Alternate method

Instead of using the (i – d) curve we suggest that using the equations

$$n^2 = \sin^2 i_2 + [\frac{\sin i_1 + \sin i_2 \cos A}{\sin A}]^2 \qquad \ldots 6$$

or alternatively the equation

$$n^2 = \sin^2 i_1 + [\frac{\sin i_2 + \sin i_1 \cos A}{\sin A}]^2 \qquad \ldots 7$$

from which the refractive index can be obtained with out much error since i_1,A and i_2 can be accurately measured. All the experiments, like the method suggested by Doughal [2, 3], necessitate high experimental reliability.

Small Angle Prism

In textbooks the equation for a small angle prism is given as

$$d = (n - 1) A \qquad \ldots 8$$

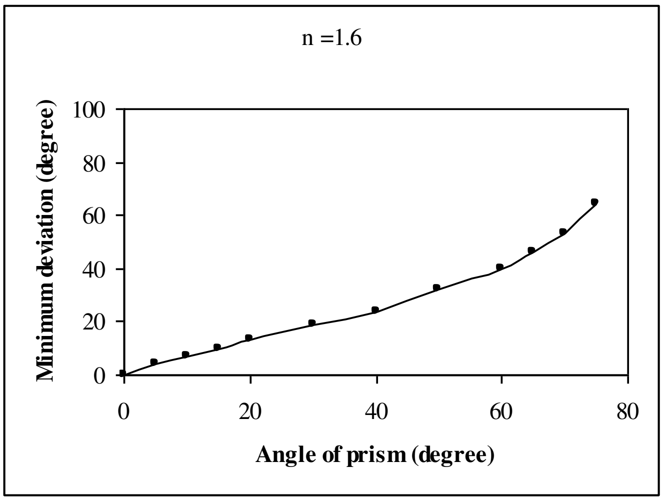

Figure 5: The (d – A) curve for n=1.6

Where d is the minimum deviation. This cannot be a general equation since it is applicable only for small angles of incidence. The general equation for a small angle prism can be obtained as given below. From (3) we get

$D = -A + 2\sin^{-1}(n \sin(\frac{A}{2}))$,

Series expansion of which gives

$D = (n-1) A + \frac{n}{24}(n^2 - 1) A3 + \text{order } (A^5) + \ldots\ldots$

Neglecting the higher order terms, this general expression reduces to

$$D = (n - 1) A \qquad \ldots 9$$

This is the most general equation which is valid for any small angle prism. This can also be verified by plotting (D - A) curve for a constant *n*.

(i-d) curve for small angle prism

The (i – d) curve for prism of different angle $A=10^0$, 8°, 5°, 2° are given in Figure-6. The angle of incidence on the right side is taken as negative and left side as positive. It is seen that deviation is constant between 2^0 and 15^0 for $A = 10^0$, between -2^0 and 15^0 for $A = 8^0$, between -5^0 and 15^0 for $A = 3^0$, between -10^0 and 15^0 for $A = 2^0$ prism. Thus for a small angle the prism deviation is constant and is equal to the D for a wide range of angle of incidence. From Figure-6 it is clear that the equation-9 is valid at least up to $A = 20^0$

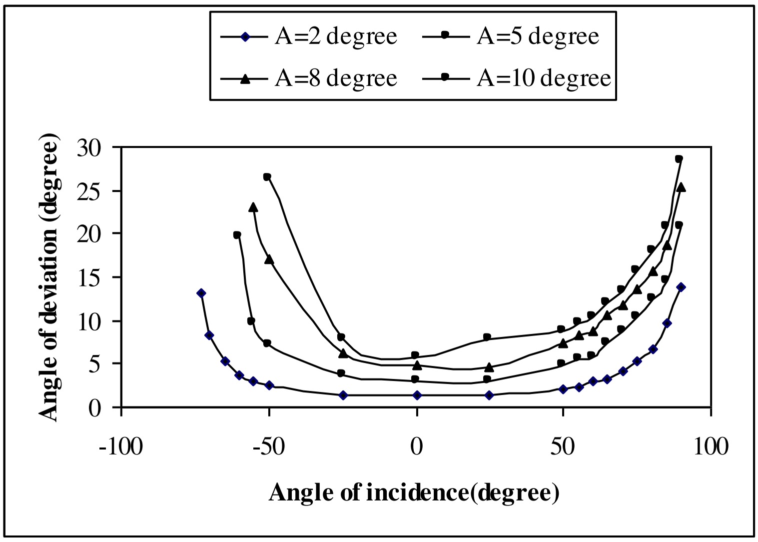

Figure 6: The (i – d) **curve for n=1.6**

Conclusions

We have critically examined the (i – d) curve method using large angle prisms particularly of 60^0 in laboratory experiments for determining the refractive index of the prism. We have clearly shown that for prisms of large angles, there can be no deviation below a certain angle of incidence. Also, we have shown that unless we know the refractive index we will not be able to calculate the minimum angle of incidence at which deviation is observable. We have shown (Figure-6) that for small angle prisms the deviation will be finite even for incident angular range of -75^0 to $+ 89^0$. While it is possible to observe the minimum deviation for small angle prisms, as seen from Figure-6 (angle of deviation remains constant over a wide range of angles of incidence), no such deviation can be observed with large angle prisms with certainty for an undergraduate experimenter unless sophisticated instruments are used. This is clear from Figure-2. Thus it has been established that small angle prisms are most suitable for determining the refractive index using (i – d) curve method. Hence conventional method of using 60^0 prisms is much inferior to small angle prisms and may therefore be discarded.

References

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