M 7589

Reg. No. : ....

Name : ....

III Semester B.Sc. Degree (CCSS-Reg./Supple./Improv.) Examination, November 2014
CORE COURSE IN STATISTICS
3B03 STA : Probability Theory

Time: 3 Hours

Max. Weightage : 30

Instruction : *Use of calculators and statistical tables are* ***permitted.***

PART – A

(Answer **any 10** questions. Weightage **one each**).

1. Define :
   1) Sample space of a random experiment
   2) Event.

2. Write down the 3rd axiom of probability. Also show that $P(\overline{A}) = 1 - P(A)$.

3. Given $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$, $P(A/B) = \frac{1}{6}$, find P(B/A).

4. The probability of Mr. A living 20 years more is $\frac{1}{5}$ and that of Mr. B is $\frac{1}{7}$. Find the probability that at least one of them will survive the next 20 years.

5. Examine whether the function $P(x) = 1/2^x$, x = 1, 2, 3, ... is a probability mass function.

6. Given the distribution function $F(x) = 1 - e^{-x}, x \geq 0$, find probability density function.

7. Define joint distribution function F(x, y) of (x, y) and state any one property of it.

P.T.O.

8. A random variable X has probability distribution.

x :	0	1	2	3
p(x) :	0.1	0.2	0.3	0.4

Find the probability distribution of $Y = X^2 + 1$.

9. Express AM, GM and HM of a random variable in terms of expectation.

10. Define cov (X, Y) and ρ (X, Y) where ρ denote correlation coefficient between X, Y.

11. Give any 2 relationships between moments and cumulants of a random variable. **(10×1=10)**

PART – B

(Answer **any 6** questions. Weightage **2 each**).

12. What are the basic assumptions of classical definition to probability of an event. List its main limitations.

13. If A and B are independent events, show that $\overline{A}$ and $\overline{B}$ are independent.

14. A random variable X has the following probability distribution.

x :	– 2	–1	0	1	2	3
p(x) :	0.1	k	0.2	2 k	0.3	3 k

1) Find k

2) Evaluate $P(-2 < X < 2)$.

15. If $f(x, y) = 4xy$, $0 < x < 1$, $0 < y < 1$, is the joint pdf of (X, Y), are X and Y independent.

16. Define mathematical expectation of a random variable. Calculate E(X) if X has pdf $f(x) = \frac{1}{4}$, – 2, < x < 2 and zero elsewhere.

17. If $f(x, y) = \frac{x + 3y}{24}$ at (1, 1), (1, 2), (2, 1) and (2, 2) and zero otherwise, examine whether E(XY) = E(X) E(Y).

18. Show that $M_{aX+b}(t) = e^{bt} M_X(at)$ where a and b are constants.

19. Find moment generating function of X if X has pdf $f(x) = \frac{1}{2} e^{-|x|}, -\infty < x < \infty$.

20. If X_1 and X_2 are independent random variables, prove that $\phi_{X_1, X_2}(t_1, t_2) = \phi_{X_1}(t_1) \cdot \phi_{X_2}(t_2)$, where ϕ denote characteristic function. **(6×2=12)**

PART – C

(Answer **any 2** questions. Weightage **4 each**).

21. State and prove Addition theorem on probability for 2 events. State the theorem in the case of 3 events.

22. Let X be a continuous random variable with

$$f(x) = \begin{cases} ax, & 0 \le x \le 1 \\ a, & 1 \le x \le 2 \\ -ax + 3a, & 2 \le x \le 3 \\ 0 & \text{elsewhere} \end{cases}$$

1) Determine constant a

2) Compute $P(X < 1.5)$, $P(X \ge 1.5)$.

23. Let (X, Y) be jointly distributed with pdf $f(x, y) = e^{-y}$, $0 < x < y < \infty$ and zero otherwise. Find (1) $f_2(y)$ (2) $f(x/y)$ (3) $E(X/Y = y)$.

24. A coin is tossed until a head appears. Let X denote the number of tosses required. Write down the probability distribution of X and determine E(X) and mode of the distribution. **(2×4=8)**