M 2336

Reg. No. : ....

Name : ....

III Semester B.A./B.Sc./B.Com./B.B.A./B.B.A.T.T.M./B.B.M./B.C.A./B.S.W./ B.A. Afsal Ul Ulama Degree (CCSS – Reg./Supple./Improv.) Examination, November 2012

CORE COURSE IN STATISTICS

3B03 STA : Probability Theory

Time : 3 Hours Max.Weightage : 30

Instruction : *Use of calculators and statistical tables **permitted**.*

PART – A

Answer **any 10** questions (Weightage **1 each**) : **(10×1=10)**

1. Define :

   i) Random experiment ii) Sample space

   iii) Sample print iv) Event

2. A coin is tossed continuously till head appears for the first time. Write down the sample space.

3. Give the aniomatic definition of probability.

4. Distinguish between pair-wise independence and total independence of three events.

5. A random variable X has the following density function $f(x) = \begin{cases} K & \text{if } 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$

   Evaluate the value of K.

6. The distribution function of a random variable X is given below :

$$F(x) = \begin{cases} 0 & \text{if } \quad x < 0 \\ \frac{1}{8} & \text{if } \quad 0 \le x < 1 \\ \frac{3}{8} & \text{if } \quad 1 \le x < 2 \\ \frac{6}{8} & \text{if } \quad 2 \le x < 3 \\ 1 & \text{if } \quad x \ge 3 \end{cases}$$

   Find $P\{1 \le x \le 2\}$.

P.T.O.

7. The following table gives the probability distribution of a random variable X.

X	:	–2	–1	0	1	2
P{X : x}	:	$3/16$	$1/16$	$1/4$	$1/8$	$3/8$

Find the probability distribution of χ^2.

8. Show that the mathematical expectation of a random variable X exists, if and only if, E |x| exists.

9. Find the mean and variance of a random variable X with probability density function.

$$f(x) = \begin{cases} 6x(1-x) & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

10. The joint density function of two random variables X and Y is given by

$$f(x, y) = \begin{cases} x + y & \text{if } 0 < x < 1,\ n < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find $E(X/Y = y)$.

11. Define the characteristic function of a random variable.

PART – B

Answer **any 6** questions (Weightage **2 each**) **(6×2=12)**

12. State and prove the addition theorem for probabilities.

13. State and Bayes theorem.

14. The probability distribution of a random variable X is given as follows :

X	:	1	2	3	4	5	6	7
P{X = x}	:	k	2k	2k	3k	k^2	$2k^2$	$7k^2 + k$

i) Find the value of k   ii) Find $P\{x \geq 6\}$ and $P\{x < 6\}$.

15. Show that the correlation coefficient between two random variables lies between –1 and 1.

16. A random variable X has the following mass function.

$$f(x) = \frac{1}{n};\ n = 0, 1, 2, \ldots n$$

Find the moment generating function of X.

17. Define cumulants. Express the first three cumulants in terms of the moments.

18. Define probability generating function (Pgt). Discuss how the mean and variance of a discrete random variable can be obtained from the pgt.

19. Express the first four central moments in terms of the raw moments.

20. Two random variables X and Y have the joint density

$$f(x, y) = \begin{cases} 8xy & \text{if } 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find :

i) $P\{x < \frac{1}{2} \cap y < \frac{1}{4}\}$

ii) The conditional densities

iii) Are X and Y independent ?

PART – C

Answer **any two** questions (Weightage **4** each) : **(2×4=8)**

21. A company produces an item in three machines A, B and C. The daily production figures of these three machines are respectively 300, 450 and 250 units. The percentage of defectives produced by these machines are 1, 2 and 7 respectively. An item is drawn at random from a days production and is formed to be defective. What is the probability that it is not produced by machine C ?

22. i) Define joint, marginal and conditional distributions of two random variables X and Y.

ii) Two random variables X and Y have the joint probability mass functions.

$$f(x, y) = \begin{cases} \frac{x^2 + y}{32}; & x = 0, 1, 2, 3 \\ & y = 0, 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the marginal distributions of X and Y. Also find the conditional distribution of X given Y = y.

23. Two random variables X and Y have the joint density

$$f(x, y) = \begin{cases} 21x^2y^3 & \text{if } 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the conditional mean and variance of X given Y = y.

24. i) If X is a continuous random variable with density function f(x) and y = g(x) is a monotonic functions of X, show that the density function of Y is given by $h(y) = f(x)\left|\frac{dx}{dy}\right|$.

ii) A random variable X has the following density function

$$f(x) = \begin{cases} \frac{1}{\pi} & \text{if } -\pi/2 < x < \pi/2 \\ 0 & \text{otherwise} \end{cases}$$

Find the density function of tan X.
