M 11244

Reg. No. : ....

Name : ....

III Semester B.A./B.Sc./B.Com./B.B.A./B.B.A. T.T.M./B.B.M./B.C.A./B.S.W. Degree (CCSS – Reg./Supple.) Examination, November 2011
CORE COURSE IN STATISTICS
3 B03 STA : Probability Theory

Time : 3 Hours Max. Weightage : 30

Instruction : Use of calculators and Statistical Tables ***permitted.***

PART – A

Answer **any 10** questions. (Weightage **1 each**) **(10×1=10)**

1. What is Statistical Regularity ?
2. Give the frequency definition of probability of an event.
3. A coin is tossed continuously till head appears for the first time. Write down the sample space.
4. State the anions of probability.
5. Distinguish between discrete and continuous random variables giving one example each.
6. A continuous random variable x has the following density function.

$$f(x) = \begin{cases} ax \text{ if } 0 < x < 1 \\ a \text{ if } 1 < x < 2 \\ -ax + 3a \text{ if } 2 < x < 3 \\ 0 \quad \text{otherwise} \end{cases}$$

Find the value of a.

P.T.O.

7. A discrete random variable has the following distribution :

   X : – 1 0 1

   P { X = x) : ¼ ½ ¼

   Find the distribution function of X.

8. Define the mathematical expectation of a random variable. Give an example of a random variable for which expectation does not exist.

9. If X and Y are two random variables, define the conditional mean and conditional variance of X given Y = y.

10. Define the raw and central moments of a random variable.

11. A random variable X has the following mass function.

   $$f(x) = \frac{1}{n} \; ; x = 1, 2, \ldots n$$

   Find the characteristic function of X.

PART – B

Answer **any 6** questions. (Weightage **2 each**) **(6×2=12)**

12. Define total independence and pair-wise independence of three events. Illustrate using an example that pair-wise independence does not imply total independence.

13. State and prove Bayes' theorem.

14. Define the distribution function of a random variable. If F(x) is the distribution function of a continuous random variable X, find the distribution of F(X).

15. If X and Y are any two random variables, prove that $(EXY)^2 \leq EX^2 \, EY^2$.

16. Two random variables X and Y have the following joint mass function :

   $$f(x, y) = \frac{2x+y}{32} \; ; \quad \begin{matrix} x = 0, 1, 2 \\ y = 0, 1, 2 \end{matrix}$$

   Find the conditional mean of X given Y = 1

17. A random variable X has the probability density function $f(x) = \frac{1}{2} e^{-|x|}$; $-\infty < x < \infty$.

Find the moment generating function of X.

18. Define cumulants. Express the first three cumulants in terms of the moments.

19. Define probability generating function. A random variable X has the following probability mass function $f(x) = (\frac{1}{2})^x$; x = 1, 2, 3, ..

Find the probability generating function of X.

20. Two random variables X and Y have the joint density

$$f(x, y) = \begin{cases} 8xy\,; & 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find:

i) $P\{X < \frac{1}{2} \cap Y < \frac{1}{4}\}$

ii) Marginal densities of X and Y.

Examine whether X and Y are independent.

PART – C

Answer **any two** questions. (Weightage **4 each**) **(2×4=8)**

21. There are three urns having the following composition of black and white balls.

Urn I : 7 white, 3 black balls

Urn II : 4 white, 6 black balls

Urn III : 2 white, 8 black balls

One of the urns is chosen at random with probabilities 0.20, 0.60 and 0.20 respectively and two balls are drawn at random without replacement from the selected urn. If the balls are found to be white and red, what is the probability that they came from urn I ?

22. Two random variables X and Y have the joint probability density function

$$f(x, y) = \begin{cases} 21\, x^2 y^3 & \text{if } 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the conditional mean and variance of X given Y = y.

23. A random variables X has the following density function :

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} \quad ; -\infty < x < \infty$$

Obtain the density function of x^2.

24. Two random variables X and Y have the following joint mass function :

$$f(x, y) = \frac{x+2y}{18}; \quad \begin{matrix} x = 1, 2 \\ y = 1, 2 \end{matrix}$$

Find the correlation coefficient between X and Y.
