M 8785

Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, November 2010
STATISTICS (Core) (Course – 3)
3B03 STA : Probability Theory

Time : 3 Hours   Total Weightage : 30

***Instruction** : Use of calculators and statistical tables permitted.*

PART – A

Answer any 10 questions. **(Weightage 1 each)**   **(10×1=10)**

1. Define the following terms :

   i) Random experiment   ii) Sample space

   iii) Sample point   iv) Event

2. Give the classical definition of probability of an event.

3. Four coins are tossed together. Write down the sample space.

4. Using the axioms of probability, prove that :

   i) $P(\phi) = 0$ ii) $P(A^C) = 1 - P(A)$ iii) $0 \le P(A) \le 1$.

5. Distinguish between discrete and continuous sample spaces giving an example each.

6. Two unbiased dice are thrown together. Obtain the probability distribution of the sum of the numbers shown by the dice.

7. A random variable X has the following probability distribution :

X	:	−1	0	1
P{X = x}	:	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

   Find the probability distributions of (i) $2X + 1$ (ii) X^2.

P.T.O.

8. Define the mathematical expectation of a random variable. Give an example of a random variable for which expectation does not exist.

9. Express the fourth central moment of a random variable in terms of the first four raw moments.

10. Two random variables X and Y have the following joint density function :

$$f(x, y) = \begin{cases} 2 \text{ if } & 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}.$$

Find the conditional mean of X given Y = y.

11. Define the characteristic function of a random variable. What is the advantage of characteristic function over moment generating function ?

PART – B

Answer **any six** questions. **(Weightage 2 each)** **(6×2=12)**

12. State and prove the addition theorem for probabilities.

13. State and prove Bayes' theorem.

14. Define the distribution function of a random variable. A discrete random variable X has the following probability distribution :

X :	1	2	3	4	5
P{X = x} :	$\frac{1}{15}$	$\frac{2}{15}$	$\frac{3}{15}$	$\frac{4}{15}$	$\frac{5}{15}$

Find the distribution function of X.

15. With usual notation, prove that

$E[E(X/Y)] = EX$

16. Two random variables X and Y have the following probability mass function :

$$f(x, y) = \begin{cases} \frac{x+2y}{18}; & x = 1, 2 ; y = 1, 2 \\ 0 & \text{otherwise} \end{cases}$$

Find the covariance between X and Y.

17. A random variable X has the density function

$$f(x) = \begin{cases} x & \text{if } 0<x<1 \\ 2-x & \text{if } 1<x<2 \\ 0 & \text{otherwise} \end{cases}$$

Find the moment generating function of X.

18. Define cumulant generating function. Explain how the cumulants are generated from the cumulant generating function.

19. Define probability generating function. A random variable X has the following mass function.

$f(x) = \left(\frac{1}{2}\right)^x$; $x = 1, 2,..$

Find the probability generating function of X.

20. Two random variables X and Y have the joint density function

$$f(x, y) = \begin{cases} 8xy & \text{if } 0<x<y<1 \\ 0 & \text{otherwise} \end{cases}$$

Find :

i) $P\{X < \frac{1}{2} \cap Y < \frac{1}{4}\}$

ii) Marginal distributions of X and Y.

Examine the independence of X and Y.

PART – C

Answer **any two** questions. **(Weightage 4 each)** : **(2×4=8)**

21. The contents of urns I, II and III are as follows :

1 white, 2 black and 3 red balls

2 white, 1 black and 1 red ball

4 white, 5 black and 3 red balls

One urn is chosen at random and two balls are drawn from it. They happen to be white and red. What are the probabilities that they came from urn I, urn II and urn III ?

22. i) Define joint, marginal and conditional distributions of two random variables.

ii) Two random variables X and Y have the following joint mass function.

$$f(x, y) = \frac{x^2 + y}{32}; x = 0, 1, 2, 3 \quad ; \quad y = 0, 1$$

Find the marginal distributions of X and Y. Also find the conditional distribution of X given Y = y.

23. Two random variables X and Y have the joint density

$$f(x, y) = \begin{cases} 21x^2y^3 & \text{if} \quad 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the conditional mean and variance of X given Y = y.

24. i) A random variable X has the density function

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{1-\frac{1}{2}x^2} \quad ; -\infty < x < \infty$$

Obtain the probability density function of X^2.

ii) If F(x) is the distribution function of continuous random variable X, find the density function of Y = F(X).

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