M 6589

Reg. No. : ....

Name : ....

II Semester B.Sc. Degree (CCSS – Reg./Supple./Improv.) Examination, May 2014
COMPLEMENTARY COURSE IN STATISTICS
(For Maths/Comp.Sc. Core)
2 C02 STA : Probability Theory and Random Variables

Time : 3 Hours | Max. Weightage : 30

***Instruction** : Use of calculators and tables are **permitted**.*

PART – A

Answer **any ten** questions. **(Wt. 1 each)**

1. Define the following terms :
   i) Sample space
   ii) Probability space
   iii) Borel field.
2. State the axioms of probability.
3. If A is any event in a sample space S show that P(A′) = 1 – P(A).
4. A coin is repeatedly tossed till a head turns up. Write down the sample space of the experiment.
5. Define conditional probability.
6. Show that for any three events A, B and C.
   P(A B C) = P(A) . P(B/A) . P(C/AB).
7. Distinguish between pairwise independence and mutual independence of three events A, B and C.
8. Define a random variable. Distinguish between discrete and continuous random variables.

P.T.O.

9. What are the properties of a probability density function ?

10. Define joint distribution function of a pair of random variables.

11. Define marginal and conditional density functions. **(10×1=10)**

PART – B

Answer **any six** questions. **(Wt. 2 each)**

12. If A and B are any two events in a sample space S. Show that
$P(A \cap B) \leq P(A) \leq P(A \cup B) \leq P(A) + P(B)$

13. If A_1, A_2, A_3 & ..... A_n are n events show that
$P(A_1 \cap A_2 \cap .... A_n) \geq P(A_1) + P(A_2) + ... P(A_n) - (n-1)$

14. Let A and B be two possible events of a random experiment with P(A) = 0.4, $P(A \cup B) = 0.7$ and P(B) = P. For what choice of P are the events A and B :
   i) disjoint
   ii) independent.

15. Give P(A) = P(B) = P(C) = 0.4, P(A B) = P(A C) = P(B C) = 0.2 and P(A B C) = 0.1. Find the probability of occurrence of
   i) atleast one of the events
   ii) exactly one of the events
   iii) exactly two of the events.

16. For three mutually exclusive and exhaustive events
$A, B, C, P(A) = \frac{1}{2} P(B) = \frac{1}{3} P(C)$. Find P(A), P(B) and P(C).

17. A continuous random variable X has the probability density function $f(x) = \frac{1}{\theta}.e^{-x/\theta}$, $x \geq 0,\ \theta > 0$.

18. For the probability mass function $f(x) = e.\left(\frac{1}{2}\right)^x$, $x = 0, 1, 2, \ldots \infty$. evaluate the constant C and find $P(x > 3)$.

19. The distribution function of a random variable X is given by

$$F(x) = 0 \text{ if } x \le 1$$
$$= k(x-1)^4 \text{ if } 1 < x \le 3$$
$$= 1 \text{ if } x > 3.$$

Find :

i) k and

ii) the probability density function of x.

20. If X has the probability density function $f(x) = e^{-x}$, $x > 0$. Obtain the probability density function of $y = e^{-x}$. **(6×2=12)**

PART – C

Answer **any two** questions.

21. State Baye's theorem. Three machines A, B, C produce 60, 30, 10 percent respectively of the total production of a factory. It is estimated that A produces 2 percent defectives, B produces 3 percent and C produces 4 percent defectives in their production. An item chosen randomly from the total production is found to be defective. What is the probability that it has come from machine A ?

22. Evaluate the distributions function F(x) for the following density function and calculate F(2)

$$f(x) = \frac{x}{3} \text{ if } 0 < x \le 1$$
$$= \frac{5}{27}(4-x) \text{ if } 1 < x \le 4$$
$$= 0 \text{ otherwise.}$$

23. Let x has the density function $f(x) = \frac{x+2}{6}$, $0 < x < 2$

$= 0$ otherwise.

Let $g(x) = 0$ if $0 < x \leq 1$

$= 1$ if $1 < x \leq 3/2$

$= 2$ if $x \geq 3/2$

Find the probability mass functions of g(x).

24. Give that $f(x, y) = k.\ e^{-x-2y}$, $x > 0$, $y > 0$

$= 0$ otherwise

where k is a constant represents a joint p.d.f. Obtain the value of the constant k and the marginal distributions of X and Y. Examine whether X and Y are independent. **(2×4=8)**
