

Reg. No. : .....

Name : .....

**V Semester B.Sc. Degree (CCSS – Reg./Supple./Imp.)**  
**Examination, November 2015**  
**CORE COURSE IN STATISTICS**  
**5B05 STA : Mathematical Analysis – I**

Time : 3 Hours

Max. Weightage : 30

## PART – A

Answer any 10 questions.

(1 each)

1. Give example (one each) for a) Monotonic increasing sequence b) Oscillating sequence.
2. Define limit of a sequence  $(a_n)$ .
3. If  $(a_n)$  converge to a limit  $l$ , show that  $(|a_n|)$  converge to  $|l|$ .
4. What is meant by sequence of partial sums of a series  $\sum a_n$ ? How will you connect convergence of a series and convergence of its sequence of partial sums?
5. Show that if  $\sum a_n$  converges,  $\lim_{n \rightarrow \infty} a_n = 0$ . Is the converse true?
6. Define conditional convergence of a series.
7. State Raabe's test for convergence of a series.
8. Define limit of a function.
9. When will you say that a function has removable discontinuity at a point?
10. Define left hand derivative of a function  $f$  at a point  $C$ .
11. State Roll's theorem.

(10×1=10)

P.T.O.



## PART – B

Answer **any 6** questions.**(2 each)**

12. Show that a sequence cannot converge to more than one limit.
13. Establish : Every monotonic sequence bounded above converges.
14. Examine the convergence of  $(a_n)$  where
- a)  $a_n = \left(1 + \frac{1}{n}\right)^n$                       b)  $a_n = \frac{1}{n!}, n \in \mathbb{N}$
15. If  $\sum a_n$  converges,  $\sum b_n$  converges, show that  $\sum a_n b_n$  converges.
16. State and prove Cauchy's root test.
17. Examine convergence of the series a)  $\sum \frac{1}{n}$  b)  $\sum \frac{1}{n^2}, n \in \mathbb{N}$ .
18. Show that a function which is differentiable at a point is necessarily continuous at that point. Is the converse true ? Justify your answer.
19. Obtain Maclaurin's expansion of  $\sin x$ .
20. State and prove Lagrange's mean value theorem of differential calculus. **(6×2=12)**

## PART – C

Answer **any 2** questions.**(4 each)**

21. State and prove Cauchy's general principle of convergence of a sequence of real numbers.
22. a) Examine convergence of the series :  $\frac{1}{1 \times 2 \times 3} + \frac{3}{2 \times 3 \times 4} + \frac{5}{3 \times 4 \times 5} + \dots$   
 b) Define absolute convergence of a series. Show that every absolutely convergent series is convergent.
23. State and prove Lagrange's mean value theorem.
24. State Taylor's theorem. Using the theorem, prove that  $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots, x \in \mathbb{R}$ .

**(2×4=8)**