M 4455

Reg. No.: ....

Name: ....

V Semester B.A./B.Sc./B.Com./B.B.A./B.B.A. T.T.M./B.B.M./B.C.A./B.S.W./ B.A. Afsal Ul Ulama Degree (CCSS – Reg./Supple./Improv.) Examination, November 2013
Core Course in Statistics
5 B05 STA : MATHEMATICAL ANALYSIS – I

Time : 3 Hours Max. Weightage : 30

PART – A

Answer **any ten** questions. **(one each)**

1. Define a bounded sequence.
2. Examine whether the following sequences are convergent or not :

   1) (a_n) where $a_n = \frac{1}{n}$

   2) (a_n) where $a_n = (-1)^n$
3. Give a necessary and sufficient condition for a monotonic sequence to be convergent.
4. If (a_n), (b_n) and (c_n) are sequences such that $a_n \leq b_n \leq c_n \; \forall_n$ and $\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = l$, then what is the value of $\lim_{n\to\infty} b_n$?
5. Evaluate $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n+1}$.
6. State Cauchy's Root Test for convergence of a positive term series.
7. Define conditional convergence of a series.
8. Evaluate $\lim_{x\to a}\frac{x^2-a^2}{x-a}$, if it exists.
9. Define continuity from left of a function.

P.T.O.

10. State Lagrange's mean value theorem.

11. Give Maclaurin's expansion of e^x, $x \in R$ with Lagrange's form of remainder.

(10×1=10 Weightage)

PART – B

Answer **any six** questions. **(two each)**

12. Establish uniqueness of limit of a convergent sequence.

13. Define monotonic increasing and monotonic decreasing sequences. Give examples.

14. Show that every convergent sequence is Cauchy.

15. If series $\sum a_n$ converges, show that $\lim_{n \to \infty} a_n = 0$.

16. State and prove Cauchy's general principle of convergence of a series.

17. Show that the series $1 + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \dots$ converges.

18. If f and g are functions such that $\lim_{x \to c} f(x) = l$, $\lim_{x \to c} g(x) = m$, show that $\lim_{x \to c} (f + g)(x) = l + m$. Write down the corresponding result for the function f – g.

19. Explain the three types of discontinuities of a function at a point.

20. Examine whether $f(x) = |x|$ is derivable at $x = 0$. **(6×2=12 Weightage)**

PART – C

Answer **any two** questions. **(four each)**

21. Distinguish between limit and limit point of a sequence. Find the limit points of (a_n) where $a_n = 1 + (-1)^n$, $n \in N$. Is it convergent ?

22. State and prove D'Alembert's Ratio Test for convergence of a positive term series. Test the behaviour of the series $\frac{x}{1} + \frac{x^2}{2} + \frac{x^3}{3} + \dots$

23. If f is continuous on a closed interval [a, b], then show that f is bounded in [a, b] and f attains its supremum of least once in [a, b].

24. State and prove Rolle's theorem. Verify the theorem in the case of the function $f(x) = 2x^3 + x^2 - 4x - 2$, $x \in \left(-\sqrt{2}, \sqrt{2}\right)$. **(2×4=8 Weightage)**

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