M 2000

Reg. No. : ....

Name : ....

V Semester B.A./B.Sc./B.Com./B.B.A./B.B.A.T.T.M./B.B.M./B.C.A./B.S.W./ B.A. Afsal Ul Ulama Degree (CCSS – Reg./Supple./Improv.) Examination, November 2012

CORE COURSE IN STATISTICS

5B05 STA : Mathematical Analysis – I

Time : 3 Hours Max. Weightage : 30

PART – A

Answer **any ten** questions. (Weightage **1 each**) : **(1 each)**

1. State a necessary and sufficient condition for the convergence of a sequence.

2. Evaluate $\lim_{n \to \infty} \frac{1+2+...+n}{n^2}$.

3. Define a sequence which is oscillating finitely.

4. Is the sequence 1, –2, 3, –4, 5, ... is monotonic and bounded ?

5. Define "Cauchy Sequence".

6. State D' Alembert's Ratio test for convergence of a series with positive terms.

7. Is the series $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+...$ convergent ?

8. State the necessary and sufficient condition for the existence of limit of a function f at a point x = C, C $\in$ R.

9. When do you say that a function f is having removable discontinuity at x = C, C $\in$R.

10. Define f′ (C), the derivative of a function f at x = C, C $\in$ R.

11. State Rolles' theorem.

(10×1=10)

P.T.O.

PART – B

Answer **any six** questions. (Weightage **2 each**)

12. Examine the behaviour of the following sequences (convergent/divergent/oscillating)
    1) $a_n = 1 + (-1)^n, n \in N$
    2) $a_n = 1 + \frac{1}{n}, n \in N.$

13. If (a_n) converges to say l, show that for each $\varepsilon > 0$, $\exists$+ ve integer m such that $|a_{n+p} - a_n| < \varepsilon \ \forall n \geq m \wedge p \geq 1.$

14. Show that a monotonic increasing sequence which is bounded above converges.

15. Define an 'Infinite series' and its convergence.

16. Discuss the convergence of the series $1 + r + r^2 + \ldots, 0 \leq r \leq 1$ [with proof].

17. State and prove limit form of comparison test for convergence of a positive term series.

18. If f and g are functions such that

$$\lim_{x \to c} f(x) = l, \ \lim_{x \to c} g(x) = m, \text{ then } \lim_{x \to c} (fg)(x) = lm.$$

19. Examine the continuity and identify the type of discontinuity of the following function at the origin.

$$f(x) = \frac{x - |x|}{x}, \ x \neq 0$$
$$= 0 \quad , x = 0$$

20. State and prove Cauchy's mean value theorem. **(6×2=12)**

PART – C

Answer **any two** questions. (Weightage **4 each**).

21. Show that (a_n) where $a_n = \left(1+\frac{1}{n}\right)^n$, $n \in N$ is convergent and that $\lim\limits_{n \to \infty} \left(1+\frac{1}{n}\right)^n$ lies between 2 and 3.

22. Define :
    1) Alternating series
    2) Absolute convergence and conditional convergence of a series.

    Also examine whether the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$ is conditionally convergent.

23. Define :
    1) continuity and
    2) uniform continuity of a function f on an interval I.

    Show that a function which is uniformly continuous on an interval is continuous on that interval.

24. State and prove Maclaurin's theorem with Lagranges' form of remainder. Hence find the Maclaurin's expansion of the function $f(x) = \log(1 + x)$, $-1 < x < 1$.

(2×4=8)

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