Reg. No. : ....

Name : ....

M 11423

V Semester B.A./B.Sc./B.Com./B.B.A./B.B.A.T.T.M./B.B.M./ B.C.A./B.S.W. Degree (CCSS – Regular) Examination, November 2011
CORE COURSE IN STATISTICS
5B05 STA : Mathematical Analysis – I

Time: 3 Hours Max. Weightage : 30

PART – A

Answer **any ten** questions. Weightage **one each** :

1. When do you say that (a_n) converge to a real number l^2 ?

2. Examine boundedness of (a_n) where $a_n = 1 + \frac{1}{2} + ... + \frac{1}{n}$.

3. Evaluate $\lim_{n \to \infty} \frac{2n^2 - 5}{3n^2 + 7n}$.

4. To which value will the sequence (x^n) converge if $|x| < 1$?

5. State the Cauchy's General Principle of convergence of a series.

6. State Cauchy's Root test for convergence of a positive term series.

7. For what values of p will the series $\Sigma \frac{1}{n^p}$ converge ?

8. Evaluate $\lim_{x \to 0-} \frac{|x|}{x}$.

9. Write down the condition under which a function f is having discontinuity of the first kind.

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10. State Darboux's theorem in Differentiation.

11. Using the concept of derivative, how will you check whether a function is increasing or decreasing.

PART – B

Answer **any six** questions. Weightage **2 each** :

12. Show that every convergent sequence is bounded.

13. If $(a_n) \to a$, $(b_n) \to b$, show that $(a_n b_n) \to ab$.

14. Define limit point of a sequence. Find the limit points of the sequence 1, 2, 1, 4, 1, 6, ...

15. If Σa_n converges, show that $\lim_{n\to\infty} a_n = 0$. Is the converse true ?

16. If Σa_n and Σb_n are two positive term series and $\exists$ a +ve integer m such that $\frac{a_{n+1}}{a_n} \leq \frac{b_{n+1}}{b_n}$ $\forall n \geq m$, show that Σa_n and Σb_n behave alike.

17. Test the convergence of the series $\Sigma \frac{n^2 - 1}{n^2 + 1} x^n$.

18. Show that $\lim_{x\to 0} \frac{e^{1/x} - 1}{e^{1/x} + 1}$ does not exist.

19. If a function f is continuous on a closed interval [a, b], show that f attains its bounds at least once in [a, b].

20. Show that if f is derivable at a point C, and $f(C) \neq 0$, then the function (1/f) is also derivable at C and $\left(\frac{1}{f}\right)'(C) = -\frac{f'(C)}{[f(C)]^2}$.

PART – C

Answer **any two** questions. Weightage **4 each** :

21. State and prove Bolzano-Weierstrass theorem for sequences.

22. State and prove logarithmic test for convergence of a series with positive terms. In which situations is this test more helpful ?

23. Define continuity of a function at a point $x = C,\ C \in R$. If $[x]$ denotes the largest integer $\leq x$, then discuss the continuity at $x = 2$ for the function, $f(x) = x - [x]\ \forall x \geq 0$.

24. Given the function $f(x) = 0,\ x \leq 0$

$$= x,\ x > 0.$$

Show that f is continuous but not derivable at $x = 0$.
