M 6064

Reg. No. : ....

Name : ....

VI Semester B.Sc. Degree (CCSS – Reg./Supple./Improv.) Examination, May 2014
Core Course in Statistics
6B10 STA : MATHEMATICAL ANALYSIS – II

Time : 3 Hours Max. Weightage : 30

PART – A

Answer **any ten** questions. **Each** question carries a weightage **1**.

1. Define refinement of a partition.
2. Define limit of a function.
3. Show that $\beta(m,n)=\beta(n,m)$.
4. State the sufficient condition for maxima and minima of the function f (x, y) at (a, b).
5. Define uniform convergence.
6. State any two properties of Riemann integral.
7. Investigate the continuity of f (x, y) at (1, 2) for $f(x, y) = x^2 + 2y \quad (x, y) \neq (1, 2)$
   $= 0 \quad (x, y) = (1, 2)$
8. Give an example of a linear vector space.
9. State the fundamental theorem of integral calculus.
10. Give an example of a bounded function which is not Riemann integrable.
11. Examine whether $\int_0^1 \sqrt{1-x^2}\,dx$ converges. **(10×1=10)**

PART – B

Answer **any 6** questions. **Each** question carries a weightage **2**.

12. State and prove the first mean value theorem of integral calculus.
13. Examine the convergence of $\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx$.

P.T.O.

14. Are the following vectors linearly independent ?

(3, 0, 2), (7, 0, 9) and (4, 1, 2).

15. Prove that a bounded function having a finite number of discontinuity points, is integrable on a closed set.

16. Define uniform convergence. Show that $f(x) = x^2$ is uniformly continuous on $(-1, 1)$.

17. Prove that every continuous function is integrable.

18. Find the relative maxima or minima of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$.

19. Show that the function f, where

$$f(x,y)=\frac{xy}{\sqrt{x^2+y^2}} \quad \text{if } x^2+y^2 \neq 0$$
$$=0 \qquad x=y=0$$

is continuous, possesses partial derivatives but is not differentiable at the origin.

20. If f and g are Riemann integrable, prove that f + g is also Riemann integrable.

(6×2=12)

PART – C

Answer **any 2** questions. **Each** question carries a weightage of **4**.

21. State and prove a necessary and sufficient condition for the integrability of a function.

22. State the sufficient conditions for maxima and minima of the function f (x) at (a, b). Find the maxima and minima of the function $x^3 + y^3 - 3x - 12y + 20$.

23. a) Explain comparison test for convergence of an improper integral.

b) If f is bounded and integrable on [a, b], then $|f|$ is also bounded and integrable on (a, b]. Prove.

24. Explain the Lagrange's method of multipliers. Find the maximum value of 8 xyz subject to the condition

$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ x > 0, y > 0 and z > 0. **(2×4=8)**

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