Reg. No. : ..............................

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M 493

**VI Semester B.A./B.Sc./B.Com./B.B.A./B.B.A.T.T.M./B.B.M./B.C.A./B.S.W./ B.A. Afsal Ul Ulama Degree (CCSS-Regular) Examination, April 2012**
**CORE COURSE IN STATISTICS**
**6B10 STA : Mathematical Analysis – II**

Time : 3 Hours  Max. Weightage : 30

PART – A

Answer **any 10** questions. **Each** question carries a weightage **1**.

1. Define neighbourhood of a point.
2. If $P^1$ is the refinement of a partition P then for a bounded function f, show that $U(P^1, f) \leq U(P, f)$.
3. State first mean value theorem.
4. What are implicit and explicit functions ?
5. Give an example of a linear vector space.
6. State the condition for maxima and minima of the function f(x, y) at (a, b).
7. Find the partial derivatives of the function $f(x, y) = x^2y + 3x^3 - 6xy + 6y^2 + 5$.
8. If $f(x, y) = \frac{xy}{x^2 + y^2} \quad (x, y) \neq (0, 0)$
   $= 0 \quad (x, y) = (0, 0)$
   check whether the function is continuous at (0, 0).
9. Examine whether $\int_0^1 \sqrt{1 - x^2}\, dx$ converges.
10. Define Beta and Gamma functions.
11. Give an example of a bounded function which is not Riemann integrable. **(Wt. 10×1=10)**

P.T.O.

## PART – B

Answer **any 6** questions. **Each** question carries a weightage **2**.

12. Establish the integrability of a continuous function.

13. State and prove the fundamental theorem of integral calculus.

14. Explain comparison test for convergence of an improper integral.

15. Examine the convergence of $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$.

16. Examine whether the function

$$f(x, y) = xy\frac{(x^2 - y^2)}{x^2 + y^2} \quad \text{if } x^2 + y^2 \neq 0$$
$$= 0 \quad x = y = 0$$

is differentiable at the origin or not.

17. Show that linear independence and dependence in a system of vectors is not affected by a scalar multiplication of vectors by non zero scalars.

18. Find the partial derivatives $f_x$ and $f_y$ if $f(x, y) = x^2y + 3x^3 - 6xy + 6y^2 + 5$.

19. Define gamma integral. Establish the relation between beta and gamma functions.

20. Show that the sequence $\{f_n\}$ where $f_n = \frac{x}{n+x}$ is uniformly convergent in (0, k).

**(Wt. 6×2=12)**

## PART – C

Answer **any 2** questions. **Each** question carries a weightage **4**.

21. If $f_1$ and $f_2$ are 2 bounded and integrable functions on (a, b) then $f = f_1 + f_2$ is also integrable on (a, b) and

$$\int_a^b f_1\,dx + \int_a^b f_2\,dx = \int_a^b f\,dx$$

Prove this statement.

22. Define linearly independent and linearly dependent vectors in a system of vectors show that the vectors (2, 3, –1, 1) (1, –1, –2, –4), (3, 1, 3, –2) and (6, 3, 0, –7) form a linearly independent set. Also express any of these vectors as a linear combination of others.

23. Explain the Lagrange's method of multipliers. Find the maximum value of 8 xyz subject to the condition

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1, \quad x>0, y>0 \text{ and } z>0.$$

24. State and prove the necessary and sufficient condition for the integrability of a function.

**(Wt. 2×4=8)**