M 26103

Reg. No. : ..............................

Name : ..................................

# Third Semester M.A./M.Sc./M.Com. Degree (Reg./Sup./Imp.) Examination, November 2014
# STATISTICS
## Paper – 3.1 : Stochastic Processes

Time: 3 Hours Max. Marks : 70

***Instructions :*** *1) Answer **any five** questions without omitting **any** Unit.*
*2) **All** questions carry **equal** marks.*

### UNIT – 1

1. a) Define a stochastic process. Explain the classification of stochastic processes based on their index set and statespace. Give one example to each category.

   b) Explain the specification of a stochastic process and state the Kolmogorov consistency theorem in this context. **(7+7)**

2. a) Define a Markov process and a process with independent increments. Bring out the mutual implications between them, if any.

   b) Define a discrete parameter Martingale. Give an example.

   c) What is a Wiener process ? Briefly discuss its applications. **(6+4+4)**

### UNIT – 2

3. a) What do you mean by a time homogeneous Markov chain ? Explain. Show that such a process is completely specified by its one-step transition probability matrix and the initial distribution.

   b) Establish the Chapman-Kolmogorov equation for a Markov Chain with stationary transition probabilities, when the time domain is discrete. Also state (without proof) the analogous version of identity in the continuous domain. **(7+7)**

P.T.O.

4. a) What is Gambler's ruin problem ? Find the probability of ultimate ruin of the Gambler, under this problem.

   b) Define recurrent and transient states of a Markov chain. Prove that state 'i' is recurrent if, and only if, $\sum_{n=1}^{\infty} P_{ii}^{n} = \infty$ (in the usual notation). **(7+7)**

UNIT – 3

5. a) Stating the postulates of a Poisson process $\{X(t), t \geq 0\}$, derive the expression for $P\{X(t) = n\}$.

   b) Show that $\{ N(t), t \geq 0 \}$ is a Poisson process if, and only if, the successive inter-arrival times form a sequence of i.i.d. exponential random variables.

6. a) Explain Birth and Death process. Obtain the limiting probabilities of the process, stating the condition for its existence.

   b) Define a compound Poisson process. Give an example. Derive the expression for the mean, variance and covariance functions of a compound Poisson process. **(7+7)**

UNIT – 4

7. a) Distinguish between strict and weak sense stationary processes, citing one example to each.

   b) Obtain the spectral representation of the covariance function of a weakly stationary process.

   c) Define moving average and auto-regressive processes. **(6+4+4)**

8. a) Define "Renewal process generated by the distribution function F" and the "Renewal function". Show that the renewal function and F determine each other uniquely.

   b) Prove that, for a renewal process $\{ N(t), t \geq 0 \}$ with mean inter-arrival time, $\mu$ finite, $\frac{N(t)}{t} \to \frac{1}{\mu}$ with probability 1 as $t \to \infty$. **(7+7)**

UNIT – 5

9. a) If $\{ X_n, n \geq 0 \}$ is a Galton-Watson Branching Process (GWBP), P(s) is the probability generating function of the offspring distribution and $P_n(s)$ is that of $X_n$, show that $P_n(s) = P_{n-1}(P(s)) = P(P_{n-1}(s))$, and hence find the mean and variance of $X_n$.

   b) Discuss the probability of ultimate extinction in the GWBP when the offspring distribution is geometric (P). **(9+5)**

10. a) Explain a continuous time branching process with general variable lifetime.

    b) If $\pi$ is the probability of ultimate extinction of a Branching Process $\{X_n, n \geq 0\}$ with $X \cdot (0) = 1$ and m is the mean of the offspring distribution then show that $\pi = 1$ if and only if $m \leq 1$. **(7+7)**

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