M 24158

Reg. No. : ....

Name : ....

Third Semester M.A./M.Sc./M.Com. Degree (Reg./Sup./Imp.) Examination, November 2013
STATISTICS
Paper – 3.1 : Stochastic Processes

Time: 3 Hours Max. Marks : 70

Instructions : *Answer **any five** questions without omitting **any** Unit. **All** questions carry **equal** marks.*

UNIT – I

1. a) Distinguish between weak and strict stationary stochastic processes. Give an example for each case.

   b) Show that a stochastic process with independent increments is Markovian.

   c) State Kolmogorov's consistency theorem and state its applications. **(6+4+4)**

2. a) When do you say that $\{X_n, n \geq 1\}$ is a martingale with respect to $\{Y_n, n \geq 1\}$? Let $\{Z_n\}$ be a sequence of independent randomariables with $E(Z_n) = 1$ for all n. Show that

$$\left\{X_n = \prod_{i=1}^{n} Z_i, n = 1, 2, \ldots\right\} \text{ is a Martingale.}$$

   b) Show that the one-step transition probabilities together with the initial probability distribution completely determines a Markov chain. **(7+7)**

UNIT – II

3. a) Distinguish between transient and persistent states in a Markov chain.

   b) Show that persistence is a class property.

   c) State and derive Chapman-Kolmogorov equations for a Markov chain $\{X_n, n \geq 0\}$. **(4+4+6)**

P.T.O.

4. a) For a Gambler's ruin problem with state space $S = \{0, 1, 2, ..., N\}$, O and N being absorbing states, determine the probability of absorption from any state i in to state N.

   b) Let $\{X_n, n \geq 0\}$ be a Markov Chain on the state space $\{-1, 0, 1\}$, initial distribution $\left(\frac{1}{3}, \frac{2}{3}, 0\right)$ and the transition probability matrix.

$$P = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

   Obtain $P[X_2 = 1]$ and $P[X_1 \neq X_2]$. **(7+7)**

UNIT – III

5. a) Define a Poisson process and a compound Poisson process. Give an example for each process.

   b) For a Poisson process $\{X(t), t \geq 0\}$, show that the number of events occuring in any interval follows a Poisson distribution. **(7+7)**

6. a) Define a birth and death process $\{X(t), t \geq 0\}$ and derive an expression for $P_n(t) = P[X(t) = n]$ under the steady state.

   b) For a Yule process $\{X(t), t \geq 0\}$ show that $P[X(t) = j \mid X(0) = i]$ can be expressed in the form of a binomial distribution. **(8+6)**

UNIT – IV

7. a) Define a renewal process $\{N(t)\}$ and renewal function, $\{M(t)\}$. Prove that $M(t) = F(t) + \int_0^t M(t-u)\, dF(u)$ under suitable conditions, where $F(\cdot)$ is the inter-arrival time distribution.

   b) Let r_t and δ_t be the excess life and the current life respectively in the context of a renewal process $\{N(t)\}$. If r_t and δ_t are independent random variables for all t then prove that $\{N(t)\}$ is a Poisson process.

8. a) Define an autoregressive model of order P and state the conditions for its stationarity.

   b) Obtain the autocorrelation function of a first order Auto Regressive process. (AR(I)).

   c) Show that an AR(I) processe is Markovian. (5+4+5)

UNIT – V

9. a) For a branching process $\{X_n\}$ with $X_0 = 1$ and $\varphi_n(t) = E(t^{X_n})$, $0 \leq t \leq 1$, establish the relation $\varphi_n(t) = \varphi_1(\varphi_{n-1}(t))$.

   b) For a branching process $\{X_n\}$ show that $P[0 < \lim X_n < \infty] = 0$.

10. a) If the offspring distribution corresponding to a branching process $\{X_n\}$ with $X_0 = 2$ is given by the pgf $\varphi(t) = (t^3 + 2t + 3)/6$, find the probability of extinction of the process.

    b) Obtain $E(X_t)$ and Var (X_t) for a continuous time branching process.
