M 18343

Reg. No. : ....

Name : ....

III Semester M.Sc. Degree Examination, November 2010
STATISTICS
Paper 3.1 : Stochastic Processes

Time : 3 Hours Max. Marks : 70

***Instructions :** 1) Answer **any five** questions without omitting any Unit.*
*2) **All** questions carry **equal** marks.*

UNIT – I

1. a) Define a stochastic process. Explain the meaning of its state space and the index set. 2

b) Describe the classification of stochastic processes based on their index set and the state space. Give an example for each case. 4

c) Distinguish between a weakly stationary and strictly stationary stochastic processes with examples. 4

d) Let k and y be independent random variables with $E(x) = \theta$, $E(y) = \beta$, $V(x) = \sigma_1^2$ $V(y) = \sigma_2^2$. Define $Z(t) = x + yt$, $t \geq 0$ is $\{ Z(t), t \geq 0 \}$ a weakly stationary stochastic process ? Justify your answer. 4

2. a) What do you mean by a process with stationary and independent increments ? Give an example for such a process.

b) Let $\{x(t), t \geq 0 \}$ with $X(0) = 0$ be a process having independent increments. Show that

i) $\{x(t), t \geq 0 \}$ is a Markov process and

ii) $Cov (x(s), x(t)) = Var (x(s))$ for $0 < s \leq t$.

c) Define a martingale. Give an example. Show that the sum of independent and identically distributed r.v.s. with mean of forms a martingale.

P.T.O.

UNIT – II

3\. a) With respect to a Markov chain : define a class property and show that define a class property and show that recurrence is a class property.

b) Show that in a finite Markov chain all states can not be transient.

c) Consider a Markov chain with state space $S = \{0, \pm 1, \pm 2, \ldots\}$ and TPM. $P = ((p_{ij}))$, given by $p_{i,i+1} = \frac{1}{3}$ and $p_{i,i-1} = \frac{2}{3}$, $p_{ij} = 0, j \neq i-1, i+1$. Check whether the Markov chain is transient.

4\. a) When do you say that a Markov chain has a stationary distribution ?

b) Examine whether a unique stationary distribution exists for a Markov chain with TPM.

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} & 0 \\ \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} \end{bmatrix}$$

c) Show that a recurrent state is visited by a Markov chain infinitely often with probability one.

UNIT – III

5\. a) Give any two definitions of a Poisson process and prove that they are equivalent.

b) Let $\{x(t), t \geq 0\}$ be a birth and death process and $p_{in}(t) = P(x(t) = n \mid x(0) = i)$. Derive the differential equations satisfied by $p_{in}(t)$.

6. a) Define a compound Poisson process and give an example to it.

 b) Find a necessary and sufficient condition for a pure birth process to converge.

 c) If $\{x(t), t \geq 0\}$ is a Poisson process, obtain the conditional distribution of x(u) given x(t) = n, when u < t.

UNIT – IV

7. a) Define renewal process and renewal function. State and prove elementary renewal theorem.

 b) Define a moving average process of order q and obtain its auto correlation function.

8. a) Find the joint distribution of r_t and δ_t, the excess life and current life respectively corresponding to a renewal process.

 b) If r_t and δ_t are independent for all $t \geq 0$, show that the process is a Poisson process.

UNIT – V

9. a) Show that the probability of extinction of a Branching Process (B.P.) is the smallest positive root of the equation $\varphi(s) = s$, where $\varphi(s)$ is the pgf of the off spring distribution.

 b) Compute the probability of extinction whose offspring distribution is geometric.

 c) Define a continuous time B.P. and explain how a linear growth birth and death process can be viewed as a B.P.

10. a) Let $\{x(t), t \geq 0\}$ be a continuous time B.P. with transition probabilities $\{p_{ij}(t)\}$. Let F(t, s) be the pgf of $\{p_{ij}(t)\}$ and u(s) be the pgf of transition densities then prove that

$$\frac{\partial}{\partial t} F(t, s) = u(s) \frac{\partial}{\partial s} F(t, s) \text{ and } \frac{\partial}{\partial t} F(t, s) = u(F(t, s)).$$

 b) For a discrete time B.P. Show that $E(X_{n+k} \mid x_n) = m^k X_n$ a.e. for k = 0, 1, 2, . . . , n = 0, 1, 2, . . . with usual notations.

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