M 16671

Reg. No. : ....

Name : ....

Third Semester M.Sc. Degree Examination, October 2009
STATISTICS
Paper – 3.1 : Stochastic Processes

Time: 3 Hours Max. Marks: 70

***Instructions :** 1) Answer **any five** questions without omitting **any** Unit.*
*2) **All** questions carry **equal** marks.*

UNIT – I

1. a) Define a Markov chain and a Markov process. Let $\{X_n\}$ be a sequence of independent binomial r.v.s. Is $\{X_n\}$ a Markov chain ? Justify your answer.

   b) How do you classify the stochastic processes based on their state space and index set ? Describe them with examples.

   c) Distinguish between a Weiner process and a Gaussian process. Are they stationary ?

2. a) Define a Poisson process and check whether it is stationary.

   b) Let $\{X_n, n = 1, 2, ...\}$ be a sequence of iid exponential r.v.s. with unit mean. Show that $\left\{ Y_n = \prod_{i=1}^{n} X_i, n = 1, 2, ... \right\}$ is a martingale.

   c) Let $\{X(t), t \geq 0\}$ with $X(0) = 0$ be a process having independent increments. Show that $\{X(t), t \geq 0\}$ is a Markov process. Is it a stationary process ?

UNIT – II

3. a) Define an irreducible Markov chain. Show that if $i \leftrightarrow j$ and i is recurrent then j is also a recurrent state.

   b) In a recurrent irreducible aperiodic Markov chain, prove that $\lim_{n\to\infty} P_{jk}^{(n)} = \lim_{n\to\infty} P_{kk}^{(n)}$, where $P_{jk}^{(n)}$ is the probability of the Markov chain moving from state j to state k in 'n' steps.

4. a) Classify the states of the Markov chain with state space {1, 2, 3, 4} and T.P.M.

$$P = \begin{bmatrix} \frac{1}{4} & \frac{1}{4} & \frac{1}{3} & \frac{1}{6} \\ 0 & \frac{2}{3} & 0 & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{4} & \frac{1}{4} \\ 0 & \frac{1}{4} & 0 & \frac{3}{4} \end{bmatrix}$$

b) If $f_{ii}^{(n)}$, the n-step first return probability for state i, is $\frac{n}{2^{n+1}}$, n = 1, 2, ... find the mean recurrence time.

c) State and derive Chapman-Kolmogorov equations for a Markov chain.

UNIT – III

5. a) Define a Poisson process. Derive the distribution of inter-occurrence times.

b) Describe a birth-death process. For this process, derive the differential-difference equations and hence derive the steady state probability distribution of system size in an M/M/1 queue.

6. a) In a continuous time Markov chain, with usual notations prove that

$$\lim_{h \to 0} \frac{1 - p_{ii}(h)}{h} = v_i \text{ and}$$

$$\lim_{h \to 0} \frac{p_{ij}(h)}{h} = q_{ij} \text{ when } i \neq j.$$

b) For the Yule process $\{X(t), t \geq 0\}$, which starts with i individuals, show that

$$p_{ij}(t) = \binom{j-1}{i-1} e^{-\lambda t i}(1 - e^{-\lambda t i})^{j-i}, \; j \geq i \geq 1.$$

UNIT – IV

7. a) Distinguish between weak stationary and strong stationary processes. Give an example for each case.

   b) Define an AR(1) model and MA(1) model. Do they provide stationary processes always ? Justify the answer.

   c) Obtain the renewal function for a renewal process $\{N(t), t \geq 0\}$ and the inter-arrival time having probability density function :

   $f(x) = \lambda^2 x e^{-\lambda x}, \; x \geq 0$.

8. a) For a Poisson renewal process, show that the current life and the excess life are independently distributed.

   b) For a renewal process, with usual notations, show that

   $E(S_{N(t)+1}) = E(X_1)\{M(t)+1\}$.

UNIT – V

9. a) Explain a Galton-Watson Branching Process (B.P.) When is a B.P. critical, sub-critical and super-critical ?

   b) If $\{X_n\}$ is a Galton-Watson B.P. and $\phi_n(s)$ is the pgf of X_n, then show that $\phi_n(s) = \phi_{n-k}(\phi_k(s))$ for all k such that $1 \leq k \leq n$.

10. a) For a B.P. show that the probability of extinction π is less than or equal to the smallest positive root of A(s) = s, where A is the generating function.

    b) Show that a B.P. is a Markov process. Derive the distribution of total number of progeny in a B.P.
