M 15144

Reg. No. : ....

Name : ....

III Semester M.Sc. Degree Examination, November 2008
STATISTICS
Paper – 3.1 : Stochastic Processes

Time : 3 Hours   Max. Marks : 70

*Instructions : 1) Answer **any five** questions **without** omitting **any** Unit.*
*2) All questions carry **equal** marks.*

Unit – I

1. a) When do you say that a stochastic process
   i) is strictly stationary
   ii) is weakly stationary
   iii) possesses stationary increments
   iv) possesses independent increments

   Give an example for a process with at least two of the above properties.

   b) State Kolmogorov's consistency theorem and explain its importance.

   c) Let $\{X(t), t \geq 0\}$ be a stochastic process such that

   $$P[X(t) = n] = \frac{t^{n-1}}{(1+t)^n}, n = 1, 2, ...$$

   Is $\{X(t), t \geq 0\}$ a stationary process ? Justify your answer.

2. a) When do you say that a stochastic process is Gaussian ? Does it have any relation with a Weiner process ? Justify your claim.

   b) Let $\{X_i, i = 1, 2, ...\}$ be a sequence of iid r.v.s. with unit mean. Then show that $\{Z_n = \prod_{i=1}^{n} X_i, n = 1, 2, ...\}$ is a Martingale.

P.T.O.

c) Let {Xn}be a sequence of iid Bernoulli r.v.s with

$P[X_n = 1] = P[X_n = -1] = \frac{1}{2}$ and define $Y_n = \sum_{i=1}^{n} X_i$, n = 1, 2, ...

show that $\{Y_n\}$ is a Martingale as well as a Markov chain.

Unit – II

3. a) Define :

   i) A Markov chain

   ii) Recurrent state

   iii) Transient state

   iv) Positive recurrent state

   v) Ergodic state and

   vi) Periodicity of a state.

   b) Show that a state i of a finite Markov chain is recurrent or transient according as $\sum P_{ii}^{(n)} = \infty$ or $\sum P_{ii}^{(n)} < \infty$.

4. a) Show that a Markov chain is completely specified when its one-step t.p.m. and initial distribution are given.

   b) Define the stationary distribution of a Markov chain. Obtain the stationary distribution (if it exists) of a Markov chain whose t.p.m. is given by

$$P = \begin{bmatrix} 0 & 2/3 & 1/3 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$

Unit – III

5. a) Define a Poisson Process $\{X(t), t \geq 0\}$ and derive its probability distribution.

   b) Consider a birth and death process $\{X(t), t \geq 0\}$ with $\lambda_n = \lambda$ and $\mu_n = n_\mu$ for all n. Derive $E[X(t) \mid X(0) = i]$.

6. a) Prove that sum of two independent Poisson process with rates λ_1 and λ_2 is again a Poisson process with rate $\lambda_1 + \lambda_2$.

   b) Derive the differential equation corresponding to a pure death process. If the process starts with i individuals find the mean and variance of Y (t), the number of individuals present at time t.

Unit – IV

7. a) Define an autoregressive model of order p and state the conditions for its stationarity.

   b) Prove that a first order autoregressive process is Markovian.

   c) Show that the renewal function is finite, non-decreasing and right continuous.

8. a) Define a renewal process $\{N(t), t \geq 0\}$ and then prove that $\frac{N(t)}{t} \to \frac{1}{\mu}$ a.s. as $t \to \infty$, where μ is the mean of the iter-arrival time.

   b) Define the renewal function. Prove in the usual notations that

   $M(t) = F(t) + \int_0^t M(t-u)\, dF(u)$, under suitable conditions (to be stated).

Unit – V

9. a) Define a discrete time branching process and discuss it with an example.

   b) For a Galton-Watson branching process $\{X_n\}$ with $P[X_0 = 1] = 1$, $E(X_1) = m$ and $V(X_1) = \sigma^2$, Obtain $E(X_n)$ and Var (X_n) ...

   c) Compute the probability of extinction of a branching process whose pgf of the off spring distribution is given by $\phi(s) = \frac{1}{4} + \frac{s}{4} + \frac{s^2}{4}$.

10. a) Show that for a continuous time branching process the probability of extinction is the smallest non-negative root of the equation u (s) = 0, where u (s) is the generating function.

    b) If $\{X(t)\}$ is a continuous time branching process, find $E(X(t))$ and $V(X(t))$.
