Reg. No. : ....

Name : ....

M 13833

Third Semester M.Sc. Degree Examination, November 2007
STATISTICS
Paper – 3.1 : Stochastic Processes

Time: 3 Hours

Max. Marks: 70

Instructions: *Answer any* ***five*** *questions without omitting any Unit.*
All questions carry ***equal*** *marks.*

UNIT – I

1. a) Define a stochastic process in general. How do you classify a stochastic process based on its index set and state space. Give example for each type.

   b) Define a martingale and explain it with an example.

   c) Can a process with independent increments be stationary ? Justify your answer with an example. (6+4+4)

2. a) What is a Weiner process ? Show that Weiner process is not strongly stationary. Is it weakly stationary ? Justify.

   b) Define a Poisson process. Show that the interval between two successive events in a Poisson process has an exponential distribution. (7+7)

UNIT – II

3. a) Define a Markov chain and its transition probability matrix (t.p.m.). Show that the n-step t.p.m. is the n^{th} power of the one-step t.p.m.

   b) Let $\{ X_n, n \geq 0 \}$ be a Markov chain with the state-space {0, 1, 2, 3} and t.p.m.

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & \frac{1}{3} & \frac{2}{3} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{2}{3} & \frac{1}{3} \end{bmatrix}$$

   Classify all the states. Find the period of state 2 and of state 3.

4. a) With respect to a Markov chain define

i) recurrent state

ii) transient state

iii) ergodic chain and

iv) communicating states.

b) Show that the state j is recurrent if and only if $\sum_{n=1}^{\infty} p_{ij}^{(n)} = \infty$, where $p_{ij}^{(n)}$ is the n-step transition probability. What is your conclusion if $\sum_{n=1}^{\infty} p_{ij}^{(n)} < \infty$? **(6+8)**

UNIT – III

5. a) Stating the postulates of a birth-death process, derive Kolmogorov's backward equations for it.

b) For a birth process with rate λ_n show that $\sum_{n=1}^{\infty} P[X(t)=n]=1$ if and only if $\sum_{n=1}^{\infty} \left(1/\lambda_n \right)$ is divergent. **(7+7)**

6. a) If T_k denotes the waiting time for the occurrence of the k^{th} event in a Poisson process then obtain the distribution of T_k.

b) Customers arrive at a service station according to a homogeneous Poisson process at the rate of 2 per minute.

i) What is the probability that no customer arrives between 8.00 a.m. and 8.05 a.m. ?

ii) What is the mean number of customers arrived during 8.00 a.m. and 12.00 noon ? **(8+6)**

UNIT – IV

7. a) What is a renewal process ? State and prove the elementary renewal theorem.

b) Describe an application of renewal processes. Clearly explain the random variables involved in your application and their importance.

8. a) Define a moving average model of order q and obtain its auto-correlation function.

b) Distinguish between weak and strong stationarity. State the conditions under which an autoregressive process of order p is stationary. (7+7)

UNIT – V

9. a) For a Galton-Watson branching process $\{X_n\}$ with

$P(X_0 = 1) = 1$, $E(X_1) = m$, $Var(X_1) = \sigma^2$, obtain $E(X_n)$ and $Var(X_n)$.

b) Show that the probability of extinction of a branching process is the smallest positive root of the equation $\phi(s) = s$, where $\phi(s)$ is the probability generating function of offspring distribution. (7+7)

10. a) Define continuous time branching process. Let F (t, s) be the pgf of $\{p_{ij}(t)\}$ and u (s) be the pgf of transition densities then prove that

$$\frac{\partial}{\partial t} F(t, s) = u(s) \frac{\partial}{\partial s} F(t, s) \text{ and } \frac{\partial}{\partial t} F(t, s) = u(F(t, s)).$$

b) For a continuous time branching process $\{X(t)\}$ derive $E\{X(t)\}$ and $Var\{X(t)\}$. (7+7)
