Reg. No. : ....

Name : ....

M 12516

Third Semester M.Sc. Degree Examination, November 2006
STATISTICS
Paper 3.1 : Stochastic Processes

Time: 3 Hours

Max. Marks: 70

*Instructions: Answer **any five** questions without omitting any Unit*
*All questions carry **equal** marks.* (14 marks)

UNIT – I

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1. a) Define :
   i) Stochastic process
   ii) State space
   iii) Index set.

   Give various classifications of stochastic processes based on nature of its state space and index set. Give suitable examples.

   b) Define processes with :
   i) Independent increments
   ii) Stationary independent increments. Consider the process $\{X(t), t \in T\}$ with $P(X(t) = n) = \frac{e^{-\lambda t}(\lambda t)^n}{n!}, \lambda > 0, n = 0, 1, 2, \ldots\ldots$. Test whether the process $\{X(t), t \in T\}$ is strictly stationary.

   c) Define Gaussian Process.

2. a) Define Markov processes and Markov chains. Let $\{X_n\}$ be a sequence of independent Poisson random variables. Is $\{X_n\}$ a Markov chain ?

   b) State Kolmogorov consistency theorem. Explain its importance.

   c) Define a Martingale.

UNIT – II

3. a) Prove that the one step transition probabilities together with the initial probability distribution completely specifies a Markov Chain.

P.T.O.

b) Specify the classes of the Markov Chain with transition probability matrix

$$P = \begin{pmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}$$ and determine whether they are transient or recurrent.

c) Define period of a state i. If $i \leftrightarrow j$, prove that i and j have the same period.

4. a) Prove that for an irreducible ergodic Markov Chain $\pi_j = \lim_{n \to \infty} P_{ij}^n$ exists and is independent of i. Furthermore, show that π_j is the unique non-negative solution of $\pi_j = \sum_{i=0}^{\infty} \pi_i P_{ij}, j \geq 0$, with $\sum_{j=0}^{\infty} \pi_j = 1$. (Ergodic Thm

b) Prove that in a finite state Markov Chain there are no null states and not all states can be transient.

UNIT – III

5. a) Describe a Poisson process and establish any two properties of it.

b) Establish a relation between Poisson process and binomial distribution.

c) Define compound Poisson process. Give its application.

6. a) Define the Birth-Death process and develop its forward equations. Also determine the limiting probabilities for a Birth-Death process.

b) Explain a linear growth process with immigration.

UNIT – IV

7. a) Define renewal process and renewal function. Prove that (under usual notations) the renewal function is given by $M(t) = \sum_{k=1}^{\infty} F_k(t)$.

b) For a renewal process, prove that $\frac{M(t)}{t} \to \frac{1}{\mu}$ where $\mu = E(X_1) < \infty$.

8. a) Define :

i) Moving average process and

ii) Auto regressive process.

Analyse for the stationarity of a moving average process.

b) In connection with a renewal process define

i) excess life

ii) current life.

Obtain the distributions of excess and current lives and their font distribution in the case when the inter renewal times have exponential distributions.

UNIT – V

9. a) Define Galton-Watson branching process. Show that if $\{X_n\}$ is a Galton-Watson branching process and $\phi_n(s)$ the probability generating function of X_n, then ϕ_n satisfies the relation $\phi_n(s) = \phi_{n-k}(\phi_k(s))$ for all k such that $1 \le k \le n$.

b) If X_n is the size of the n^{th} generation find $E(X_n)$ and $V(X_n)$.

10. a) Describe continuous time branching process and obtain the ultimate probability of extinction for a continuous time branching process.

b) Consider a discrete time branching process $\{X_n\}$ with probability generating function $\phi(s) = \frac{1-(b+c)}{1-c} + \frac{bs}{1-cs}, 0 < c < b+c < 1$,

where $\frac{1-b-c}{c(1-c)} > 1$. Assume $X_0 = 1$. Determine the conditional limit distribution $\lim_{n \to \infty} P(X_n = k \mid X_n > 0)$.