Third Semester M.Sc. Degree Examination, November 2005
STATISTICS (Paper – 3.1)
Stochastic Processes (2004 Admn.)

Time : 3 Hours Max. Marks: 70

Instructions : 1) Answer any five questions without ***omitting*** *any Unit.*
2) ***All*** *questions carry* ***equal*** *marks.*

UNIT – I

1. a) Define a stochastic process. Classify the stochastic processes in terms of nature of its state space and indexing set. Give examples for each type.

   b) Define processes with stationary independent increments. If a process $\{ X_t, t \in T \}$, where $T = [\, 0, - 0)$ has stationary independent increments and has a finite mean, then show that $E(X_t) = m_0 + m_1 t$ where $m_0 = E(X_0)$ and $m_1 = E(X_1) - m_0$.

2. a) Distinguish between strictly stationary and covariance stationary processes. Comment on the statement "There are covariance stationary processes that are not stationary".

   b) Define Markov process and Markov chain. Give examples. Identify a sequence of random variables that do not form a Markov chain.

UNIT – II

3. a) Derive Chapman - Kolmogorov equation.

   b) Describe Gambler's ruin problem. Find the probability of ultimate ruin of the gambler.

4. a) Define recurrent and transient states. Prove that all states in an equivalence class are all either recurrent or transient.

   b) Define stationary distribution of a Markov chain. Prove that ergodicity is a necessary and sufficient condition for the existence of stationary distribution, in case of an irreducible aperiodic chain.

UNIT – III

5. a) Define Yule process. Associate with a Yule process derive the expression (Under certain assumptions)

$$P_{Nn}(t) = \binom{n-1}{n-N} e^{-N\beta t} \left(1 - e^{-\beta t}\right)^{n-N} \quad ; n = N, N+1, \ldots$$

b) Given that x(t) = n, show that the n arrival times $S_1, S_2, \ldots, S_n$ have the same distribution as the order statistics corresponding to a sample of n observations taken from the uniform distribution on [0, t].

6. a) Define compound Poisson process. Give examples. Compute Cov (X(s), X(t)) for a compound Poisson process.

b) Explain birth and death processes. Give their applications in queueing models. Show that the backward equations for the birth and death process are

$$P_{oj}(t) = \lambda_0 \, [P_{ij}(t) - P_{oj}(t)]$$

$$P_{ij}(t) = \lambda_i P_{i+1,j}(t) + \mu_i P_{i-1,j}(t) = (\lambda_i + \mu_i) P_{ij}(t), \; i > 0$$

UNIT – IV

7. a) Define Moving average process. Show that this is covariance stationary.

b) Define a p^{th} order autoregressive process. Prove that an autoregressive process can be represented by an moving average process of an infinite order (under suitable conditions).

8. a) Define the renewal function M(t) and show that it satisfies the integral equation

$$M(t) = F(t) + \int_0^t M(t-y)\, dF(y), \; t \geq 0.$$

b) State and prove elementary renewal theorem.

UNIT – V

9. a) Describe Galton-Watson branching process. Show that for a Galton-Watson branching process, the probability of ultimate extinction is one if the mean number of offsprings is less than one.

b) In a Galton-Watson process show that $\{W_n\}$ where

$$W_n = \frac{X_n}{m^n} \quad (\varepsilon(X_n) = m^n) \text{ is a martingale.}$$

10. a) Describe continuous time branching process. Obtain expressions for E(x(t))and V(x(t)) , where X(t) is a continuous time branching process and X(0) = 1.

b) Obtain the probability of ultimate extinction in the case of continuous time branching process.
