Second Semester M.Sc. Degree Examination, May 2007
STATISTICS
Paper -- 2.2 : Sampling Theory

Time: 3 Hours

Max. Marks: 70

Instructions: 1) Answer **any five** questions *without omitting any unit.*
2) **All** questions carry **equal** marks.

UNIT – I

1. a) Briefly explain the census method and sample survey methods pointing out their merits and demerits.

   b) Distinguish between SRSWR and SRSWOR. Prove that in SRSWR(N, n) scheme, both $\bar{y}$ (the mean based on all units at all the draws) and $\bar{y}_v$ (the mean of the distinct units) are unbiased for the population mean $\bar{Y}$ and $V(\bar{y}_v) \leq V(\bar{y})$.

2. a) Derive an unbiased estimator of $V(\bar{y})$ in SRSWOR (N, n) scheme.

   b) Explain the method of determining the sample size in simple random sampling so as to achieve the desired level of confidence in the cases of estimating population mean and population proportion.

UNIT – II

3. a) Explain stratified sampling. Obtain an unbiased estimate of the population mean under stratified sampling scheme and derive expression for its variance under Neymann allocation.

   b) Describe systematic sampling method. Compare the efficiencies of the mean of a systematic sample with the mean of a simple random samples.

4. a) In stratified random sampling if the cost function is $C = c_0 + \sum c_h n_h$, explain how do you determine n_h such that the variance of the estimator $V(\bar{y}_s)$ is minimum subject to a total fixed cost c.

b) Consider a systematic sample from a population with linear trend $y_i = a + b_i$, $i = 1, 2, \ldots, N$ (a, b > 0 constants). Show that stratified random sampling is effective than systematic sampling.

UNIT – III

5. a) In PPSWr sampling show that the Hansen – Hurwitz estimator $\hat{Y}_{pps} = \dfrac{1}{n} \sum\limits_{i=1}^{n} \dfrac{y_i}{p_i}$

is an unbiased estimator for Y and obtain its variance.

b) What is Hurvitz – Thompson estimator ? Show that it is unbiased for any sampling design and find its variance.

6. a) Explain Lahiri's method of selecting a PPS sample pointing out its merits.

b) Compare ppswr scheme with srswr scheme and estimate the gain in precesion in ppswr - sampling over srswr.

UNIT – IV

7. a) What are Ratio estimators ? Show that in SRSWOR (N, n), the mean square error of $\hat{R}$ is approximately given by

$$E(\hat{R} - R)^2 \sim \frac{1-f}{n} R^2 \left[C_{yy} + C_{xx} - 2f C_x C_y \right]$$

Where $C_{xx} = C_x^2 = \dfrac{S_x^2}{\overline{X}^2}$, $C_{yy} = C_y^2 = \dfrac{S_y^2}{\overline{Y}^2}$

b) What are unbiased ratio-type estimators ? Discuss how a biased ratio estimator is modified to an unbiased one and obtain an unbiased estimator of population total.

8. a) What are Difference estimators ? Show that in Srswor (N, n), the estimator $\hat{y}_0 = \overline{y} + k (\overline{X} - \overline{x})$ is an unbiased estimator of $\overline{Y}$ and find its variance.

b) Critically make a comparative study of Regression estimator with Ratio estimator.

UNIT – V

9. a) In an Srswor of n clusters from a population of N clusters each contains Melements, show that the sample mean per element is an unbiased estimator of population mean per element and find its variance.

   b) If M_i is the size of the ith cluster $i = 1, 2, \ldots , N$ and $\overline{M} = \sum_{i=1}^{N} \frac{M_i}{N}$ check

   whether $\hat{\overline{\overline{Y}}} = \frac{1}{n\overline{M}} \sum_{i=1}^{n} M_i \overline{Y}_i$ where $\overline{Y}_i = \sum_{j=1}^{M_i} \frac{Y_{ij}}{M_i}$ is unbiased for $\overline{\overline{Y}}$ and derive

   an expression for its variance.

10. a) In two stage sampling with srswor at both stages suggest an unbiased estimator of the population total and find its variance.

   b) Write explanatory notes on :
   i) Double sampling for Ratio estimation
   ii) Double sampling for Regression estimator.

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