M 24421

Reg. No. : ....

Name : ....

I Semester M.A./M.Sc./M.Com./M.Sc.Computer Science Degree (Reg./Suple./Imp.) Examination, November 2013
STATISTICS
Paper – 1.1 : Probability Theory – I

Time : 3 Hours   Max. Marks : 60

Instructions : *1) **All** questions carry **equal** marks.*
*2) Answer **any one** question from **each** Unit.*

UNIT – I

I. a) Define limit inferior, limit superior and limit of sequences of sets. Show that for every sequence of sets {An}.

$\underline{\lim}\, An \subseteq \overline{\lim}\, An$.

b) Describe the following terms and illustrate it through an example.

i) Probability space   ii) Events

II. a) Define Borel σ – field in two different ways and prove their equivalence.

b) Prove or disprove the following statements.

i) Intersection of σ – fields is again a σ – field

ii) A field of subsets of Ω is also a σ – field of subsets of Ω.

UNIT – II

III. a) Distinguish between finite and σ – finite measures.

b) State and prove any two properties of probability measure.

IV. a) Let $\Omega = \{0, 1, 2, ...\}$, $\mathbb{F} = \mathbb{P}(\Omega)$, the power set of Ω. Define μ on $\mathbb{F}$ such that

$$\mu(A) = \sum_{x \in A} \frac{e^{-x}}{x!} \; \forall A \in \mathbb{F}.$$

Check whether μ is a probability measure.

b) Let X ~ N (0,1) and

$An = \{w \in \Omega \mid 0 < x(w) < n\}$. Find $\lim_{n \to \infty} P(An)$.

P.T.O.

UNIT – III

V. a) Define random variable. Give an example. If X and Y are random variables, show that X+Y is also a random variable.

b) If {Xn} is a sequence of random variables on a probability space $(\Omega, \mathbb{F}, P)$. Show that lim Xn is also a random variable on $(\Omega, \mathbb{F}, P)$, if the limit exists.

VI. a) Define distribution function. List out all properties of distribution function.

b) Let $F(x) = \begin{cases} 0 & \text{if} \quad x < -1 \\ \frac{1+x}{2} & \text{if} \quad -1 \le x < 0 \\ 3/4 & \text{if} \quad 0 \le x < 1 \\ 1 & \text{if} \quad x \ge 1 \end{cases}$

Check whether F is a distribution function ? If yes decompose F into discrete and continuous parts.

UNIT – IV

VII. a) State and prove monotone convergence theorem.

b) Beginning with the mathematical expectation of simple random variables describe the systematic development leading to expectation of arbitrary random variables.

VIII. a) State and prove dominated convergence theorem.

b) If X and Y are random variables on a probability space such that $X \ge Y$ as show that $E(X) \ge E(Y)$.

UNIT – V

IX. a) Define absolute continuity and singularity of measures. Illustrate it through examples.

b) State Radon-Nikodym theorem. Describe its application in probability. What is Radon-Nikodym derivative ?

X. a) Define the following terms :

i) Product space ii) Product σ – field iii) Product measure

b) State Fubinis theorem. Explain how this is helpful to probabilists.