M 22387

Reg. No. : ....

Name : ....

I Semester M.A./M.Sc./M.Com. Degree (Reg./Sup./Imp.) Examination, November 2012
STATISTICS
Paper – 1.1 : Probability Theory – I

Time : 3 Hours Max. Marks : 60

Instructions : ***All*** *questions carry* ***equal*** *marks.*
Answer ***any one*** *question from* ***each*** *Unit.*

UNIT – I

I. a) Let $\Omega = \{1, 2, 3, ...\}$ and $IF = IP(\Omega)$, the power set of Ω. Define : (5×12=60)

$$IF = \left\{ A \subseteq \Omega \,\middle|\, \begin{array}{l} \text{either } A \text{ is finite or} \\ A^c \text{ is finite} \end{array} \right\}$$

Check whether IF is

i) a field in Ω.

ii) a σ – field in Ω.

b) Define set function. Describe how it is different from point function ? Define additive and subadditive set functions. Give example to each.

II. a) Show that a field is a σ – field if and only if it is a monotone class.

b) Define Borel σ – field and Borel sets. Check whether the following sets in $\mathbb{R}$ are Borel sets, justify your answer.

i) $\{x \in \mathbb{R} \mid x^2 - 2x + 1 = 0\}$

ii) $\{x \in \mathbb{R} \mid a < x \leq b\}$

iii) $\left\{ x \in \mathbb{R} \,\middle|\, \begin{array}{l} x \text{ is a prime number} \\ \text{and } 0 < x < 20 \end{array} \right\}$.

P.T.O.

UNIT – II

III. a) Define measure. Distinguish between finite and σ–finite measures. Give example to each of your answer.

b) Define Lebesgue measure in $\mathbb{R}$. Find the Lebesgue measure of the following measurable sets in $\mathbb{R}$.

i) Q, the set of rationals in $\mathbb{R}$.

ii) $(1,2] \cup \{3, 7) \cup (5,8)$

iii) $(-5,5) \cap [0, 7)$.

Show that Lebesgue restricted to [0, 1] is a probability measure.

IV. a) Define probability space. If $\{A_n\}$ is a sequence of events on a probability space, show that lim inf A_n, lim sup A_n and lim A_n whenever it exists are all events on the same probability space.

b) If $\{A_n\}$ is a sequence of events such that $A_n \rightarrow A$, show that $P(A_n) \rightarrow P(A)$.

UNIT – III

V. a) Show that every non-negative random variable can be expressed as the limit of a monotone increasing sequence of simple random variables.

b) State and prove the result in connection with decomposition of a distribution to its discrete part and continuous part.

VI. a) If X is a random variable on $(\Omega, \mathbb{F}, P)$, describe what is the probability space induced by X ? Illustrate it through a simple example.

b) Define independence of random variables. If X and Y are independent random variables and g(.) a continuous real valued function, show that g(X) and g(Y) are independent.

UNIT – IV

VII. a) Define Lebesgue integral of a measurable function. Enumerate its Properties.

b) If X and Y are two measurable functions such that $X \leq Y$ and Y is integrable, show that X is also integrable.

VIII. a) State and prove Faton's lemma.

b) If X and Y are two random variables such that E(X) and E (Y) exist, show that E (X + Y) exists and E ($\alpha X + \beta Y$) = α E (X) + β E(Y) for every $\alpha, \beta \in \mathbb{R}$.

UNIT – V

IX. a) What is Radon-Nikodym derivative. Describe its properties.

b) State and prove Lebesgue decomposition theorem.

X) a) Define independence of events. Show that any subclass of an independent class of events is again independent. Show that pairwise independence does not imply mutual independence.

b) Define conditional probability measure. Illustrate it through an example.

c) State Fubini's theorem. Describe its importance in probability.
