M 20482

Reg. No. : ...............................

Name : ...................................

# I Semester M.A./M.Sc./M.Com. Degree (Reg./Sup./Imp.) Examination, November 2011
## STATISTICS
## Paper – 1.1 : Probability Theory – I

Time: 3 Hours Max. Marks : 60

***Instructions :*** *All questions carry **equal** marks. Answer **any one** question from **each** Unit.*

UNIT – I

I. a) Define field and $\sigma$-field. Show that every finite field is a $\sigma$-field.

b) Show that every monotone sequence of sets is always convergent. Identify the limit.

II. a) Define Borel $\sigma$-field in $\mathbb{R}$. Show that the set of rationals in $\mathbb{R}$ is a Borel set.

b) Define generated $\sigma$-field over a given class. If $\mathcal{C} = \{A_1, A_2, ..., A_n\}$ is a partition of $\Omega$, find the $\sigma$-field generated by $\mathcal{C}$. **(1×12=12)**

UNIT – II

III. a) Define Lebesgue measure in $\mathbb{R}$. Find the Lebesgue measure of the following sets in $\mathbb{R}$.

i) Q, the set of rationals in $\mathbb{R}$

ii) $\{x \in \mathbb{R} \mid x^2 - 4x + 4 = 0\}$

iii) $(1, 3) \cup \{4\} \cup [4, 6]$

b) Describe probability space. Give an example. What you mean by discrete probability space ?

IV. a) Let $(\Omega, \mathcal{F}, P)$ be a probability space. Prove that

i) $P(B \setminus A) = P(B) - P(A)$ whenever $A \subset B$

ii) $P\left(\bigcup_{1}^{\infty} A_i\right) \leq \sum_{1}^{\infty} P(A_i)$

where $\{A_i\}$ is a sequence of events.

b) State and prove the continuity property of probability measure. **(1×12=12)**

UNIT – III

V. a) Define measurable function. Give an example. If f and g are measurable functions, show that $\alpha f + \beta g$ is also a measurable function for every $\alpha, \beta \in \mathbb{R}$.

b) State and prove the Jordan decomposition theorem on distribution functions.

VI. a) Describe Lebesgue-Stieltje's measure. Verify that Lebesgue measure and probability measure are particular cases of Lebesgue-Stieltje's measure.

b) If $Z = (X, Y)'$ is a two-dimensional random vector, define the distribution function of Z. What are its properties ? **(1×12=12)**

UNIT – IV

VII. a) Prove that a measurable function f is integrable over a measurable set E if and only if $|f|$ is integrable over E.

b) Show that the function

$f : [0, \infty) \to \mathbb{R}$

defined by

$$f(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

is not Lebesgue integrable over $[0, \infty]$.

VIII. a) State and prove Fatou's lemma.

b) If X and Y are random variables on a probability space $(\Omega, \mathcal{F}, P)$ such that E(X), E(Y) exist, show that E(X + Y) exists and E(X + Y) = E(X) + E(Y). **(1×12=12)**

## UNIT – V

IX. a) Define Radon-Nikodym derivative. Let $\mu$ and $\nu$ be $\sigma$-finite measures such that $\mu << \nu$ and $\nu << \mu$. Show that

$$\left[\frac{d\nu}{d\mu}\right] = \left[\frac{d\mu}{d\nu}\right]^{-1}$$

b) If $f(x, y) = \frac{xy}{(x^2 + y^2)^2}$, show that

$$\int_{-1}^{1}\int_{-1}^{1} f(x, y)\, dx\, dy = \int_{-1}^{1}\int_{-1}^{1} f(x, y)\, dy\, dx.$$

X. a) Describe independence of

i) a class of events

ii) two classes of events

iii) two random variables.

b) Show that any subclass of an independent class of events is again independent. Also show that pairwise independence does not imply mutual independence.

**(1×12=12)**

---