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M 16931

Reg. No. : ...$\mathcal{B}$ A9PSST1008...

Name : ....

First Semester M.Sc. Degree Examination, November 2009
Paper – 1.1 : STATISTICS
Probability Theory

Time: 3 Hours Max. Marks: 60

Instructions : All questions carry ***equal*** *marks. Answer* ***any one*** *question from* ***each*** *Unit.*

UNIT – I

1. a) Let $\Omega = [0\ \infty)$ and F_1 be the class of all intervals of the type [a, b) or $[a, \infty)$ where $0 \le a < b < \infty$. Let F_2 be the class of all finite disjoint unions of intervals of F_1. Show that F_1 is not a field where as F_2 is a field but not a σ field.

   b) Define the limit of a sequence $\{A_n\}$. Let $\Omega = R^2$ and A_n = interior of a circle with centre $\left(\frac{(-1)^n}{n}, 0\right)$ and radius unity. Find lim sup An, lim inf An and lim An if it exists. **(6+6)**

2. a) Explain a Borel field on **R** and show that it is the σ field generated by the class of all intervals of the form (a, b) (a < b), a, b$\in$**R**.

   b) i) Show that the intersection of an arbitrary number of σ fields is a σ field.

   ii) Prove that σ field is a monotone field and conversely. **(6+6)**

UNIT – II

3. Let μ be a σ-finite measure on $\mathcal{B}(\mathbb{R})$ if $b \in \mathbb{R}$ define

   $\mu_b(A) = \mu(A - b)$ for $A \in \mathcal{B}(\mathbb{R})$, where $A - b = \{u - b / U \in A\}$. Show that

   1) μ_b is a σ finite measure on $\mathcal{B}(\mathbb{R})$.

   2) If λ denotes Lebesgue measure, then $\lambda_b(c, d] = \lambda(c, d])$ for any $c < d$, and conclude that $\lambda = \lambda_b$ for any $b \in \mathbb{R}$.

P.T.O.

4. a) Define a probability space. Given $\Omega = [0, \infty)$, $\mathcal{F}_c$ the power set of Ω and $P(A) = \sum_{K \in A \cap N} \frac{1}{2^K}$, $A \in \mathcal{F}_1$ where $N = \{1, 2, ...\}$. Show that $(\Omega, \mathcal{F}_c, P)$ is a probability space.

b) Let $\{A_n\}$ be a sequence of events. Then if $A_n \to A$ show that $P(A_n) \to P(A)$ as $n \to \infty$. **(6+6)**

UNIT – III

5. a) Show that inverse mappings preserves all set operations.

b) Define convolution of two distribution functions. Show that it is also a distribution function.

6. a) If $\{X_n\}$ is a sequence of random variables on $(\Omega, \mathcal{A})$, define $\lim_{n \to \infty} X_n$. When will you say that $X_n \to X$ uniformly on A ?

b) For a sequence $\{X_n\}$ of random variables prove that $\overline{\lim}\, X_n$, $\underline{\lim}\, X_n$ and $\lim X_n$ (if existing) are extended random variables. **(6+6)**

UNIT – IV

7. a) Let f be a Bevel measurable function on $(\Omega, \mathcal{B}, \mu)$. When do you say f is integable. If $\int f d\mu$ exists and $c \in R$ then show that $\int cf d\mu$ exists and is equal to $c \int f d\mu$.

b) Let $\{f_n\}$ be a sequence of non-negative measurable functions which convergs a.e. on a set $E \in \mathcal{B}$ to a measurable function f. Then prove that $\int_E f\, d\mu \leq \int_E f_n\, d\mu$.

8. a) Define expectation of an arbitrary random variable. Show that

$$E|X| \leq 1 + \sum_{n=1}^{\infty} P[1 \times 1 \geq n]$$

b) State and prove the monotone convergence theorem. **(6+6)**

UNIT – V

9. a) Define conditional expectation $E^{\mathscr{B}}X$ of X given $\mathscr{B}$. If $|X_n| \le Y$ prove that $X_n \xrightarrow{a.s} X \Rightarrow E^{\mathscr{B}}X_n \to E^{\mathscr{B}}X$ a.s.

b) What is Ra don-Nikodym derivative ? If λ and μ are σ finite measures on $\mathscr{A}$ and if μ is λ continuous and x is $\mathscr{A}$ – measurable whose integral $\int x d\mu$ exists then show that for every $A \in \mathscr{A}$.

$$\frac{d\phi}{d\lambda} = \frac{d\phi}{d\mu} \cdot \frac{d\mu}{d\lambda} \text{ a.s.}$$ **(6+6)**

10. a) State Fubini's theorem. Explain its implication in probability theory.

b) State the Lebesgue decomposition theorem.

c) Conditional distribution function of X given Y = y is uniform over (–y, y) and pdf of Y is $\frac{1}{y^2}$, $Y \ge 1$. Examine whether $E(E(X/Y)) = E(X)$. **(3+3+6)**