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M 15467

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First Semester M.Sc. Degree Examination, December 2008
STATISTICS
Paper – 1.1 : Probability Theory – I

Time: 3 Hours. Max. Marks: 60

*Instructions : All questions carry **equal** marks.*
*Answer **any one** question from **each** Unit.*

UNIT – I

1. a) Prove that intersection of two σ field is a σ field while union of two σ field is not necessarily a σ field.

   b) Define partition of a set. If $\{A_1, \ldots A_m\}$ and $\{B_1, B_2, \ldots B_n\}$ are two partitions of Ω. Show that $\{A_i \cap B_j\}$ is a partition of Ω. **(6+6)**

2. a) Explain how a limit of an arbitrary sequence of sets is defined.

   b) Explain a Besel field in $\mathcal{R}$ and show that it is the σ field generated by the class of all intervals of the form (a, b) (a < b), a, b $\in \mathcal{R}$. **(6+6)**

UNIT – II

3. a) Let μ be a σ a finite measure on $\mathcal{B}(\mathcal{R})$. If $b \in \mathcal{R}$ define $\mu_b(A) = \mu(A - b)$ for $A \in \mathcal{B}(\mathcal{R})$ where $A - b = \{ u - b \mid u \in A \}$. Show that

   1) μ_b is a σ finite measure on $\mathcal{B}(\mathcal{R})$

   2) If λ denotes Lebesgue measure, then $\lambda_b(c, d] = \lambda((c, d])$ for any $c < d$, and conclude that $\lambda = \lambda_b$ for any $b \in \mathcal{R}$. **(6+6)**

P.T.O.

4. a) Define a probability space. Explain how the induced probability space is obtained through a random variable. Give an example.

b) Let $\Omega = \{w_1, w_2, w_3, w_4\}$ and let $P(\{w_1\}) = \frac{1}{6}$, $P(\{w_2\}) = \frac{1}{3}$, $P(\{w_3\}) = \frac{1}{5}$, $P(\{w_4\}) = \frac{3}{10}$.

Define $A_n = \begin{cases} \{w_1, w_2\} & \text{if } n \text{ is odd} \\ \{w_2, w_3\} & \text{if } n \text{ is even} \end{cases}$

Find $P(\underline{\lim}\, A_n)$, $P(\overline{\lim}\, A_n)$, $\underline{\lim}\, P(A_n)$, $\overline{\lim}\, P(A_n)$. (6+6)

UNIT – III

5. a) Show that | X | is measurable with respect to A if X is measurable with respect to A. Is the converse true ? Justify.

b) Let X be a random variable with distribution function df

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{1}{2} & \text{if } x = 0 \\ \frac{1}{2} + \frac{x}{2} & \text{if } 0 < x < 1 \\ 1 & \text{if } x \geq 1 \end{cases}$$

Decompose F. (6+6)

6. a) Prove that the σ field generated by the inverse image of a class $\mathcal{R}$ is same as the inverse image of the σ field generated by $\mathcal{R}$.

b) Let Y : R $\to$ R and Y be defined as

$$Y(w) = 0 \text{ if } w < 5$$
$$= 1 \text{ if } w \geq 5$$

Find the σ field induced by Y. (6+6)

UNIT – IV

7. a) State and prove the monotone convergence theorem.

b) Let EX, EY and EX + EY exist. Prove that E (X + Y) = EX + EY. **(6+6)**

8. a) Let $\{f_n\}$ be a sequence of non-negative measurable functions and $f_n \to f$ a.e. on E. Then prove that $\int_E f \, d\mu \le \underline{\lim} \int_E f_n \, d\mu$.

b) Define expectation of a complex random variable Z. Show that $|EZ| \le E|Z|$. **(6+6)**

UNIT – V

9. a) Let X be $\mathcal{A}$-measurable function whose integral exists. Let $\mathcal{B} \subset \mathcal{A}$ be a σ field. Then define :

i) $E^{\mathcal{B}}X$, the conditional expectation of X given $\mathcal{B}$.

ii) The conditional probability of A given $\mathcal{B}$. $(P^{\mathcal{B}}(A))$.

b) Prove that :

i) $0 \le P^{\mathcal{B}}A \le 1$ a.s $P^{\mathcal{B}}$

ii) $A_1 \subset A_2 \Rightarrow P^{\mathcal{B}}A_1 \le P^{\mathcal{B}}A_2$ a.s $P^{\mathcal{B}}$

iii) If $A_1, A_2, ...$ are mutually exclusive sets of $\mathcal{A}$

$$P^{\mathcal{B}}\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P^{\mathcal{B}} A_i \text{ a.s } P^{\mathcal{B}}.$$ **(6+6)**

10. a) What is the Radon-Nikodym Derivative ? Show that the Radon-Nikodym derivative is unique up to sets of P-measure zero.

b) What do you understand by product-measure space ? State Fubini's theorem. **(6+6)**