Reg. No. : ....

Name : ....

M 13933

First Semester M.Sc. Degree Examination, November 2007
STATISTICS
Paper – 1.1 : Probability Theory – I

Time: 3 Hours Max. Marks: 60

*Instructions : All questions carry **equal** marks. Attempt **any five** without omitting **any** unit.*

UNIT – I

1. a) Define $\underline{\lim}$, $\overline{\lim}$ and the limit of a sequence of sets. Obtain the $\underline{\lim}\, A_n$, $\overline{\lim}\, A_n$ and the $\lim A_n$ if it exists when

$$A_n = \left\{x : 0 < x < 2 + (-1)^n/n\right\}, n = 1, 2, ...$$

b) Show that intersection of arbitrary number of sigma fields is again a sigma field. (7+5)

2. a) Define a Borel field. Show that the sigma field generated by the class $\{[a, b), a < b, a, b \in R\}$ of intervals is the Borel field.

b) If $\{A_1, A_2, A_3\}$ is a partition of Ω then obtain the minimal sigma field generated by this partition. What are the steps to be followed for generating a minimal sigma field from an arbitrary class of sets, which is not a partition ? (5+7)

UNIT – II

3. a) Define a measure. Distinguish between finite and sigma finite measures through examples. If μ is a finite measure with $\mu(\Omega) = M < \infty$ then show that $\frac{\mu}{M}$ is a probability measure. 6

b) Let $\{A_n, n \geq 1\}$ be a monotone sequence of events and $A = \lim\limits_{n \to \infty} A_n$. Then show that $P(A) = \lim\limits_{n \to \infty} P(A_n)$, where P is a probability measure. 6

P.T.O.

4. a) If $A_1, A_2 ..., A_n$ are arbitrary events then show that

$$P(A_1 \cap A_2 \cap ... \cap A_n) \geq \sum_{i=1}^{n} P(A_i) - (n-1)$$

b) Prove that probability function defined on a class $\{(a, b], a < b, a, b \in R\}$ of intervals can be uniquely extended to the minimal field containing all the intervals. (6+6)

UNIT – III

5. a) Define a random variable. Show that Borel function of $\mathcal{A}$–measurable function X is $\mathcal{A}$– measurable and induces a sub-sigma field of that induced by X.

b) Show that the class of real random variables is equivalent to the class of finite limits of sequences of simple random variables. (6+6)

6. a) Show that the inverse mapping preserves all set operations.

b) Define the distribution function of a random variable. State and prove Jordam decomposition theorem. (6+6)

UNIT – IV

7. a) State and prove Lebesgue bounded convergence theorem.

b) Let $\{U_n\}$ be a sequence of non-negative measurable functions. Then show that (7+5)

$$\int \sum_{n=1}^{\infty} U_n = \sum_{n=1}^{\infty} \int U_n$$

8. a) State and prove Fatou's lemma.

b) Define the expectation of an arbitrary real random variables (rv). For a non-negative rv X with finite expectation prove that $E(X) = \int_0^{\infty} (1 - F(x))\, dx$, when F (x) is the distribution function of X. (6+6)

UNIT – V

9. a) When do you say that a measure is absolutely continuous with respect to another measure ? Illustrate with examples.

b) State and prove Lebesgue decomposition theorem. (5+7)

10. a) Define conditional expectation of a r.v. given a σ-field. If X is a $\mathcal{B}$-measurable r.v. then show that E (XY | $\mathcal{B}$) = XE (Y | $\mathcal{B}$).

b) State and prove the smoothing property of conditional expectations. (6+6)

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