③

M 12700

Reg. No. : ....

Name : Jayanic...........

First Semester M.Sc. Degree Examination, November 2006
STATISTICS (Paper No. 1.1)
Probability Theory – I

Time: 3 Hours　　　　　　　　　　　　　　　　　　Max. Marks: 60

***Instructions** : 1) All questions carry **equal** marks.*
*2) Attempt **any five** without omitting **any** Unit.*

UNIT – I

1. a) Distinguish between a field, sigma field and Borel field. Show that a sigma field is a field. Is the converse true ? Justify your answer.

   b) Given a class $\{A_i, i = 1, 2, ...n\}$ of n sets show that there exists a class $\{B_i, i = 1, 2, ...n\}$ of disjoint sets such that $\bigcup_{i=1}^{n} A_i = \bigcup_{i=1}^{n} B_i$. (7+5)

2. a) Define a monotone field. Prove that A is a sigma field if and only if it is a monotone field.

   b) Prove or disprove :

   i) Union of two sigma fields is again a sigma field.

   ii) Intersection of two sigma fields is not a sigma field.

UNIT – II

3. a) Define outer measure and show that it is translation invariant. State the axioms of a measure.

   b) Let (Ω, A, P) be a probability space and $\{A_n\}$ be a sequence of events from A such that $\lim_{n\to\infty} A_n$ exists. Show that $P(\lim A_n) = \lim P(A_n)$. Continuity property. (6+6)

4. a) For any two events A and B show that
   $P(AB^c \cup BA^c) = P(A) + P(B) - 2P(AB)$.

   b) Define conditional probability measure and show that it satisfies the axioms of the probability measure.

   c) Show that a random variable induces a probability space. (4+4+4)

P.T.O.

UNIT – III

5. a) Define a measurable function. Show that a simple function $X = \sum_{i=1}^{n} a_i I_{A_i}$ is measurable if and only if $A_1, A_2 ....A_n$ are measurable sets.

   b) Prove that a non-negative random variable x can be written as the limit of a sequence of non-negative, non-decreasing simple random variables. (6+6)

6. a) If f_1 are f_2 are two measurable functions then show that $f_1 + f_2$ is also measurable.

Re b) Define a distribution function and state its properties. Show that every distribution function has a countable set of discontinuity points. (6+6)

UNIT – IV

7. a) State and prove monotone convergence theorem.

   b) Let f be a bounded function defined on a measurable set of finite measure. Prove that f is Lebesgue integrable if and only if it is measurable. (6+6)

8. a) Let $\{f_n\}$ be a sequence of non-negative measurable functions and $f_n \to f$ a.e. on E. Then prove that $\int_E f\, d\mu \le \underline{\lim} \int_E f_n\, d\mu$.

   b) Define the expectation of a non-negative random variable x. If X and Y are two non-negative random variables with finite expectations then show that E (X + Y) = E (X) + E (Y). (6+6)

UNIT – V

Re 9. a) State Radon-Nikodym theorem and explain its uses in statistics. Define product measure space.

Re b) Define conditional probability and conditional expectations. Show that the conditional probability obays the axioms of probability measure.

   c) Show that $E((X + Y) \mid \mathscr{B}) = E(X \mid \mathscr{B}) + E(Y \mid \mathscr{B})$. (4+4+4)

10. a) State Fubini's theorem and explain its use in probability theory.

   b) If $\mathscr{B}_1 \subset \mathscr{B}_2$ then show that $E[(X/\mathscr{B}_2)/\mathscr{B}_1] = E(X/\mathscr{B}_1)$ a.s.

   c) If $A_1, A_2, ...$ are mutually exclusive events from a sigma field A then show that $P\left[\bigcup_{i=1}^{\infty} A_i \middle| \mathscr{B}\right] = \sum_{i=1}^{\infty} P(A_i \mid \mathscr{B})$ a.s. (4+4+4)

$P[\cup_{i=1} A_i / B] = P(\cup B \cap \cup A_i) / P(B) = P(\cup B \cap A_i)/P(B) = \sum_{i=1} P(B \cap A_i)/P(B) = \sum_{i=1} P(A_i/B)$