M 10986

First Semester M.Sc. Degree Examination, November 2005
STATISTICS
Paper - 1.1 : Probability Theory – I

Time: 3 Hours

Max. Marks: 60

Instructions : All questions carry equal marks. Attempt any five without omitting any Unit.

UNIT – I

1. a) Define limit of a countable sequence of sets $\{A_n\}$.

When $A_n = A$ n=1, 3, 5

$= B$ n=2, 4, 6

Show that $\overline{\text{Lim}}\, A_n = A \cup B$ and $\underline{\text{Lim}}\, A_n = A \cap B$. When does Lim A_n exist ?

b) Let $\Omega = \{a, b, c, d\}$, $\mathcal{A} = \{\phi, \Omega, \{a, b\}, \{c, d\}\}$ $X(a) = X(b) = -1$, $X(c) = 1$, $X(d) = 2$. Examine whether X is $\mathcal{A}$ - measurable.

2. a) Let $A = \{A_1, A_2, A_3, A_4\}$ be a partition of Ω. Then write down the σ field generated by A.

b) If $\{A_n\}$ is accountable sequence with $A_n \to A$ then prove that $A_n^c \to A^c$.

UNIT – II

3. a) If $A_n \to A$, then prove that $P\, A_n \to P(A)$.

b) Let $\Omega = \{w_1, w_2, w_3, w_4\}$ and let $P(w_1) = 1/6$, $P(w_2) = 1/3$, $P(w_3) = 1/5$, $P(w_4) = 3/10$ Let $A_n = \{w_1, w_2\}$ if n is odd

$= \{w_2, w_3\}$ if n is even find $P(\underline{\text{Lim}}\, A_n)$, $P(\overline{\text{Lim}}\, A_n)$,

$\underline{\text{Lim}}\, P\, A_n$, $\overline{\text{Lim}}\, PA_n$.

4. a) Give the axiomatic definition of probability. Prove that

$$P\left(\bigcup_{i=1}^{n} A_i\right) = \sum P(A_i) - \sum_{i<j=1}^{n} P(A_i A_j) \ldots (-1)^{n-1} \sum_i \sum P(A_i A_j A_k \ldots A_l)$$

b) Describe the (i) Conditional probability measure (ii) Generalized probability measure.

UNIT – III

5. a) Show that inverse mapping preserves all set operations.

   b) Prove that random variables induces a probability space.

6. a) Let X be a mapping defined by $X : \Omega_1 \to \Omega_2$ show that

   i) If $\mathscr{A}$ is a σ field an Ω_1, then the class $\mathcal{B}$ of all sets whose inverse images beloging to $\mathscr{A}$ is also a σ field.

   ii) If $\not{c}$ is a σ field on Ω_2 then $X^{-1}(\not{c})$ is a σ field on Ω_1.

   iii) $\sigma\{X^{-1}(\not{c})\} = X^{-1}\{\sigma(\not{c})\}$.

UNIT – IV

7. A bounded function f defined on a measurable set E of finite measure is Lebesgue integrable if and only if f is measurable.

8. a) Let ϕ and ψ be simple functions which vanish outside a set of finite measure then prove that

   a) $\int a\,\phi + b\,\psi = a \int \phi + b \int \psi$ for all reals a and b

   b) if $\phi \geq \psi$ a. e then $\int \phi \geq \int \psi$.

   b) State and prove the monotone convergence theorem.

UNIT – V

9. Define conditional Expectation $E^{\mathcal{B}}$. Prove that

   i) If $\mathcal{B}$ and $\mathcal{B}_X$ are independent $E^{\mathcal{B}}X = E\,X$ a. s.

   ii) If X is $\mathcal{B}$ measurable then $E^{\mathcal{B}}XY = X\,E^{\mathcal{B}}\,Y$ a. s.

   iii) If $\mathcal{B} \subset \mathcal{B}'$ then $E^{\mathcal{B}}(E^{\mathcal{B}'}X) = E^{\mathcal{B}}X = E^{\mathcal{B}'}(E^{\mathcal{B}}X)$a.s.

10. Define conditional probability $(P^{\mathcal{B}}A)$. Prove that :

   i) $O \leq P^{\mathcal{B}}A \leq 1$ a. s.

   ii) $A_1 \subset A_2 \Rightarrow P^{\mathcal{B}}A_1 \leq P^{\mathcal{B}}A_2$, a.s

   iii) If A_1, A_2 ,.......... are mutually exclusive sets of $\mathscr{A}$

   $$P^{\mathcal{B}}\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P^{\mathcal{B}}A_i \quad a.\ s\,(P^{\mathcal{B}}).$$

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