Reg. No. : ....

Name : ....

II Semester M.A./M.Sc./M.Com. Degree (Reg./Sup./Imp.) Examination, March 2014
STATISTICS
Paper – 2.1 : Probability Theory – II

Time: 3 Hours Max. Marks : 70

Instructions : *1) Answer **any five** questions without omitting **any** Unit.*
*2) **All** questions carry **equal** marks.*

UNIT – I

1. a) Obtain Liapanov's inequality as a consequence of Jensen's inequality.

   b) State and prove Holder's inequality. **(7+7)**

2. a) State and prove Borel-Cantelli lemma.

   b) Let X_1, X_2, Y be independent and Y takes the values ± 1 with probability $\frac{1}{2}$ each. Let $Z_1 = YX_1$, $Z_2 = YX_2$. Show that Z_1 Z_2 are dependent but Z_1^2 and Z_2^2 are independent. **(7+7)**

UNIT – II

3. a) Show that if $\phi_n(u)$, n = 1, 2, ... are ch. fns. of r.v.s., then $\Sigma\lambda_n\phi_n(u)$ is a ch.fn. of a r.v., if $\lambda_n \geq 0$, $\Sigma\lambda_n = 1$.

   b) Show that for any ch.fn. ϕ, $\int\limits_{|x| \leq 4^{-1}} x^2\, dF(x) \leq \frac{3}{4^2}\{1 - Re(\phi(u))\}$. **(7+7)**

4. a) State and prove uniqueness theorem on ch.fns.

   b) If ν_r, the r^{th} absolute moment of F(x) is finite, then show that the ch.fn. is differentiable r times and $\phi^{(r)}(u) = \int (ix)^r e^{iux} dF(x)$. **(7+7)**

P.T.O.

UNIT – III

5. a) Show that $X_n \xrightarrow{P} 0$ if and only if $E\left(\frac{|X_n|}{H|X_n|}\right) \to 0$, as $n \to \infty$.

   b) If $X_n \xrightarrow{P} X$, then show that there exists a sub-sequence $\{X_{nk}\}$ of $\{X_n\}$ which converges a. s. to X. **(7+7)**

6. a) Show that $X_n \xrightarrow{r} X \Rightarrow X_n \xrightarrow{P} X$. If X_n's are a.s. bounded show that $X_n \to X \Rightarrow X_n \xrightarrow{r} X$, for all r.

   b) State and prove Helly-Bray lemma. **(7+7)**

UNIT – IV

7. a) If $X_1, X_2, \ldots$ are independent r.v.s such that $\max_{1 \le k \le n} \int_{|x| \ge A} |x| \, dF_K(x) \to 0$ as $A \to \infty$, where $F_K(x)$ is the distribution function of X_K, then show that weak law of large number holds for $\{X_K\}$.

   b) Let $\{X_n\}$ be i.i.d. with $P\left(X_n = (-1)^{k-1}k\right) = \frac{6}{(\pi k)^2}$, $k = 1, 2, \ldots$. Putting $S_n = \Sigma X_i$, show that $\frac{S_n}{n} \to b \ln 2 / \pi^2$. But strong law of large numbers does not hold. **(7+7)**

8. a) If X_K's are independent and integrable with $|X_K| \le C \le \infty$, then show that for every $\in > 0$, $1 - \frac{(\in + 2c)^2}{\sum_1^n \sigma_{K^2}} \le P\left\{\max_{k \le n} |S_k - ES_K| \le \in\right\} \le \frac{1}{\in^2} \sum_1^n \sigma_{K^2}$.

   b) State Glivenko-Cantelli theorem and describe the applications of the same. **(9+5)**

UNIT – V

9. a) Describe the applications of Central Limit Theorem.

   b) State and prove Liapanov's Central Limit Theorem.

   c) Explain multivariate Central Limit Theorem. **(4+7+3)**

10. a) Let $\{X_n\}$ be independent with $P(X_n = n\alpha) = \frac{1}{4} = P(X_n = n\alpha), P(X_n = 0) = \frac{1}{2}$ (α is a constant). Prove that Lindeberg condition holds for $\{X_n\}$.

    b) Show that product of finite number of infinitely divisible ch.fns. is infinitely divisible.

    c) Examine the infinite divisibility of the binomial distribution with p.m.f.

    $P(X_n = k) = \binom{n}{k} p^k (1-p)^{n-k}$, $0 < p < 1$, $k = 0, 1, 2, \ldots, n$. **(6+4+4)**