M 23118

Reg. No. : ....

Name : ....

II Semester M.A./M.Sc./M.Com. Degree (Reg./Sup./Imp.) Examination, March 2013
STATISTICS
Paper – 2.1 : Probability Theory – II

Time : 3 Hours Max. Marks : 70

Instructions : *1) Answer **any five** questions **without** omitting **any** Unit.*
*2) **All** questions carry **equal** marks.*

UNIT – I

1. a) State and prove basic inequality.

   b) If $E|X|^r < \infty$, then show that $E|X|^{r'} < \infty$ for $0 < r' \leq r$ and $E X^k$ exists and is finite for $k < r$, k an integer. **(7+7)**

2. a) State and prove Borel zero-one law.

   b) State that the sequence $\{X_n\}$ is a martingale sequence if and only if $E(X_n/X_1, X_2, ..., X_m) = X_m$ a.s. $(m < n)$. **(7+7)**

UNIT – II

3. a) State and prove inversion theorem on characteristic functions.

   b) Obtain the probability density function corresponding to the characteristic function $\phi(u) = e^{-|u|}, u \in R$. **(8+6)**

4. a) Show that characteristic function is continuous.

   b) If ϕ is a ch.fn., then show that Re (ϕ) is also a ch. fn.

   c) Show that under certain conditions (to be stated) the sequence of moments determine the distribution uniquely. **(5+4+5)**

UNIT – III

5. a) If $X_n \xrightarrow{P} X$ and $f(\cdot)$ is a continuous real valued function then show that $f(X_n) \xrightarrow{P} f(X)$.

b) Justify the following statement using appropriate examples. Almost sure convergence neither implies nor implied by r^{th} mean convergence. (7+7)

6. a) State and prove Helly-Bray Theorem.

b) State and prove continuity theorem on characteristic functions. (7+7)

UNIT – IV

7. a) State and prove Khintchine's Weak law of large numbers.

b) State and prove Kolmogrov's strong law of large numbers for i.i.d. r.v.s. (7+7)

8. a) State Kolmogrov's three series theorem. Describe the applications of the same.

b) Describe law of iterated logarithm.

c) State Glivenko-Cantelli theorem and describe the applications of the same. (5+4+5)

UNIT – V

9. a) State and prove Lindeberge-Feller Central Limit theorem.

b) What is Central Limit theorem ? Describe the applications of the same.

c) Explain multivariate Central Limit theorem. (7+4+3)

10. a) Let $\{X_n\}$ be independent such that

$P(X_n = n^{\alpha}) = \frac{1}{2n^{\alpha}} = P(X_n = -n^{\alpha})$, $P(X_n = 0) = 1 - n^{-\alpha}$. For $0 < \alpha < 1$, establish Liapanov's Central Limit theorem for $\{X_n\}$.

b) Define infinite divisibility of random variables. Examine the infinite divisibility of gamma distribution.

c) Give an example of random variable which is not infinitely divisible. Establish your claim. (6+4+4)

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