M 17522

Reg. No. : ....

Name : ....

Second Semester M.Sc. Degree Examination, March 2010
STATISTICS
Paper – 2.1 : Probability Theory – II

Time: 3 Hours Max. Marks: 70

Instructions : *1) Answer **any five** questions without omitting **any** Unit.*
*2) **All** questions carry **equal** marks.*

UNIT – I

1. a) State and prove the C_r – inequality.

   b) State Holder's inequality. Using the Holder's inequality prove that $\log E|x|^r$ is a convex function for $r > 0$. 10

   c) State a prove the Liapuwov's inequality. (5+2+7)

2. a) Difine Tail σ field and Tail random variable.

   b) Prove that tail functions are degenerate (a.s).

   c) State and prove the Borel 0 – 1 law. (2+5+7)

UNIT – II

3. a) Define characteristic function and establish uniqueness theorem.

   b) What do you mean by conjugate pairs of distributions ? Give any one example.

   c) What is a non-negative definite function ? Prove that a characteristic function is non-negative definite. (7+3+4)

4. a) If ϕ is a characteristic function of a random variable X prove that

   i) ϕ is continuous.

   ii) $|\phi(\cup)| \leq \phi(0) = F(+\infty) - F(-\infty), \phi(-\cup) = \overline{\phi}(\cup)$.

   iii) Characteristic function of a + bx is $e^{ina}\ \phi(b\cup)$. 7

   iv) $\overline{\phi}$ is the ch. for g – x, ϕ is real iff x is symmetric about the origin.

P.T.O.

b) Check whether the following functions are characteristic functions.

1) $\dfrac{1}{2-\phi(\cup)}$ where $\phi(\cup)$ is a characteristic function

2) $\cos^2\cup + i\sin^2\cup$. **(7+7)**

UNIT – III

5\. a) When do you say that two sequences are $\{x_n\}$ and $\{x_n^1\}$ are convergence equivalent. prove the following result.

b) Let $S_n = \Sigma X_k$ and $S_n^1 = \Sigma x_k^1$. If $\sum\limits_n P\left[x_n \neq x_n^1\right] < \infty$ then $\{x_n\}$ and $\{x_n^1\}$ are tail equivalent $\{s_n\}$ and $\{s_n^1\}$ are convergence equivalent. Also $\{s_n / b_n\}$ and $\{s_n^1 / bn\}$ $bn \uparrow \infty$ converge on the same event to the same limit.

c) If $\{x_n\}$ is a sequence of mutually indpt r-vs and let $\sigma_k^2 = \text{van } X_k$. Prove that if $\sum\limits_k \sigma_k < \infty$, then $\Sigma(X_k - Ex_k)$ converges in probabilities. **(2+5+7)**

6\. a) When do you say a sequence $\{x_n\}$

i) Converges in law to a r.v. X

ii) Converges in probability to a r.v.X.

b) Prove that $X_n \xrightarrow{P} x \Longrightarrow X_n \xrightarrow{L} X$. Prove the conditions under which the converse is also true.

c) If $X_n \xrightarrow{P} X$, prove that there exist a subsequence $\{X_{nK}\}$ of (X_n) which converges a.s to X. **(2+5+7)**

UNIT – IV

7\. a) State and prove the kolmogrov SLLN for the iid case.

b) For what values of α does the SLLN hold for an independent sequence $\{x_k\}$ with $P\left[X_K = \pm K^\alpha\right] = \dfrac{1}{2}$? **(7+7)**

8. a) Define an Empirical distribution function. What is it used to estimate ?

b) State and prove the Glivenko-Cantelli theorem.

c) Write an explanting note on the law of iterated Logarithms. **(2+7+5)**

UNIT – V

9. State and prove the Luide berg-Felle r central limit theorem and hence deduce the Lia pounov and Lindeberg – heavy forms of central limit theorems. **14**

10. a) When do you say that a distribution function F is infinite divisible ?

b) Let X be a random variable with distribution function.

$$F(x) = \begin{cases} 0 & x < -1 \\ \dfrac{x+1}{2} & -1 \leq x \leq 1 \\ 1 & x \geq 1 \end{cases}$$

Is F infinitely divisible ?

c) If $\phi(t), t \in R$ is a characteristic function and $\alpha > 0$, show that $g(t) = e^{\alpha(\phi(t)-1)}$ is characteristic function and also infinite divisible. **(2+5+7)**