M 15934

Reg. No. : ABPS STI004

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STATISTICS
Paper – 2.1 : Probability Theory – II

Time: 3 Hours Max. Marks: 70

***Instructions :** 1) Answer any five questions without omitting **any Unit**.*
*2) All questions carry **equal** marks.*

UNIT – I

1. a) State and prove the basic inequality and hence deduce the Markov inequality.
   b) When do you say
      i) Events of a class are mutually independent
      ii) Classes are independent
      iii) Random variables are independent.
   c) If X and Y are two independent random variables prove that f (x) and g(y) are also independent random variables where f and g are continuous functions of X and Y respectively. (7+3+4)

2. a) Define a Martingle. Let X be any r. v whose expectation exists and $\{Y_n\}$ be an arbitrary sequence. Prove that $Z_n = E[X \mid Y_1 Y_2 \ldots\ldots Y_n]$ is a martingle.
   b) State and prove Liapunov's inequality. (7+7)

UNIT – II

3. a) Define characteristic function of a random variable. If ϕ is the characteristic function of a random variable prove that
      i) ϕ is continuous
      ii) $|\phi(v)| \leq \phi(0) = F(+\infty) - F(-\infty)$
      iii) $\bar{\phi}$ is the characteristic function of -X. Further ϕ is real iff X is symmetric about the origin.
   b) State and prove the uniqueness theorem of characteristic functions. (7+7)

P.T.O.

4. If ν_r, the r^{th} absolute moment of F(x) is finite prove that characteristic function is differentiable r times and $\phi^{(r)}(v) = \int (ix)^r e^{ivx} dF(x)$. Also prove the converse namely if $\phi^{(r)}(0)$ exists and is finite.

$\nu_{r'} < \infty$ for $r' < r$ if r is odd and for $r' < r$ if r is even. **14**

UNIT – III

5. a) Define : i) Convergence in r^{th} mean
ii) Convergence in probability.

b) Show that $X_n \xrightarrow{r} X \Rightarrow X_n \xrightarrow{p} X$. When does the converse hold if ever it holds ?

c) Prove that $X_n \xrightarrow{r} X \Rightarrow E|X_n|^r \to E|X|^r$. **(3+7+4)**

6. a) Define convergence is distribution for a sequence of random variables.

If $X_n \xrightarrow{L} X$ and $Y_n \xrightarrow{L} C$ prove that

i) $X_n + Y_n \xrightarrow{L} X + C$

ii) $X_n Y_n \xrightarrow{L} CX$

b) State and prove the Helly-Bray Theorem. **(7+7)**

UNIT – IV

7. a) Let $\{X_n\}$ be a sequence of iid random variables and be $Y_n = X_n$ if $|X_n| \le n$.
$= 0$ otherwise

Show that $\{X_n\}$ and $\{Y_n\}$ are tail equivalent whenever $E|X_1| < \infty$.

b) If $\{X_n\}$ is a sequence of independent r.v's when do you say the SLLN holds. Also if $P\left[X_n = n^{\lambda}\right] = P\left[X_n = -n^{\lambda}\right] = \frac{1}{2}$, for what λ does the SLLN hold. **(7+7)**

8. a) Prove the following result :

Let $\{X_i\}$ be a sequence of iid r.v's with common distribution function F. A necessary and sufficient condition for the existence of $\{\mu_n\}$ such that

$$\frac{S_n}{n} - \mu_n \xrightarrow{P} 0 \text{ is that}$$

$$\text{Lim } n P[|X| > n] = \lim n[1 - F(n) + F(-n)] = 0$$

If this condition is satisfied $\mu_n = \int_{-n}^{+n} x \, dF$

b) Write explanatory notes on :

1) Law of iterated logarithm.

2) Any two application of convergence of random variables in statistics. (7+7)

UNIT – V

9. State and prove the Lindeberg-Feller CLT and hence deduce the Lypounov and Lindeberg-Levy forms of CLT. 14

10. a) Define an infinite divisible distribution. If ϕ is an infinite divisible characteristic function prove that ϕ has no real zeros.

b) If $\phi(t)$, $t \in R$ is a characteristic function and $\alpha > 0$ show that $g(t) = e^{\alpha(\phi(t)-1)}$ is a characteristic function and is also infinite divisible. (7+7)