Reg. No. :................................

Name :....................................

M 11986

Second Semester M.Sc. Degree Examination, May 2006
STATISTICS
Paper – 2.1 : Probability Theory – II

Time: 3 Hours Max. Marks: 70

Instructions : 1) Answer any five questions, without omitting any Unit.
2) All questions carry equal marks.

UNIT – 1

1. a) State and prove the Jensen's inequality. Give an example of a random variable for which the inequality becomes equality.

   b) Obtain the distribution for which the r^{th} cumulant is $K_r = p(r-1)\,!$.

2. a) What are tail events ? What can you say about the probabilities of tail events of a sequence of independent random variables ? Let $\{A_n\}$ be a sequence of independent events with $P(A_n) = \frac{1}{2}$ for all n, obtain $\overline{\lim}\, A_n$ and $\underline{\lim}\, A_n$. Examine whether $\{A_n\}$ converges.

   b) Define a martingale. Give an example of a martingale sequence. State and prove a necessary and sufficient condition for a sequence $\{X_n\}$ to be a martingale sequence.

UNIT – 2

3. a) State the inversion theorem. If the characteristic function ϕ a distribution function F is absolutely integrable, show that F is absolutely continuous with

   p.d.f. $f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-itx}\phi(t)\,dt$

   b) If $\phi(t) = e^{-t^2}$, obtain the density function.

P.T.O.

4. a) If $\phi(.)$ is a characteristic function, show that

i) $\phi(.)$ is continuous

ii) $\phi(-u) = \overline{\phi(u)}$, where $\overline{\phi(u)}$ is the complex conjugate of $\phi(u)$. Also show that if the r^{th} absolute moment is funite and the characteristic function is differentiable r times,

$$\phi^{(r)}(u) = \int (ix)^r e^{iux} \, dF(x).$$

b) State Bochner's theorem and discuss its uses. Also describe the properties of a moment sequence.

UNIT – 3

5. a) Prove that $X_n \xrightarrow{as} X$ if and only if as $m \to \infty$

$$P\left\{ \bigcup_{k=n}^{\infty} \left[w : |X_n(w) - X(w)| \geq \frac{1}{r} \right] \right\} \to 0 \text{ for all r, an integer.}$$

b) Define the convergence in distribution of a sequence of random variables. What is its role in statistics ? Give an example to show that convergence in distribution does not imply convergence of moments.

6. a) Define convergence in probability of a sequence of random variables $\{X_n\}$. If f(x) is a continuous real valued function and $X_n \xrightarrow{P} X$, then show that $f(X_n) \xrightarrow{P} f(X)$.

b) State and prove the Slustky theorem. Explain, in brief, the utility of the theorem.

UNIT – 4

7. a) Distinguish between weak and strong law of large numbers.

b) Examine whether the law of large numbers hold for the sequence of independent random variables {xk} with distribution

$$P\{Xk = \pm 2^k\} = \frac{1}{2} \; ; \; k \geq 1$$

c) Denote by $\sigma_k^2 = V(Xk)$. Show that the weak law of large numbers holds for a sequence of independent random variables $\{Xk\}$ if $\sum_{1}^{n} \sigma_k^2 / n^2 \to 0$.

8. a) State and prove Kolmogrov's three series theorem.

b) If the mutually independent random variable X_k have a common distribution $\{f(x_j)\}$ and if $\mu = E(X_k)$ exists, then show that the strong law of large numbers applies to the sequence $\{X_k\}$.

UNIT – 5

9. a) What is a central limit Problem ? State and prove the Lindberg-Levy form of the central limit theorem.

b) Let $\{X_n\}$ be a sequence of independent random variables such that X_K is normal with mean zero and variance $\frac{1}{2^n}; n \geq 1$. Show that $\{X_n\}$ obeys the C.L.T., but the Lindberg-Teller condition fails.

10. Write short notes on:

i) Infinitely divisible distribution

ii) Multivariate central limit theorems

iii) Implications between the Liapnov, Lindberg-Levy and Lindberg-Feller conditions for the C.L.T.

iv) Law of iterated logarithms.
