Name Name ....

Reg. No. ....

M 10382

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, MAY 2005
(2004 ADMISSION)

STATISTICS

PAPER 2.1 - PROBABILITY THEORY II

Time : 3 Hours

Max. Marks: 70

Probability Theory - II

All question carry EQUAL marks

Unit - 1

1. a) State and prove Holder's inequality for expectation of random variables. Deduce Schwarz's inequality.

   b) When will you say that a function $f(x)$ is convex ? If x is a random variable with $E(x) = \mu$ and $f(x)$ is a convex function, show that

   $E[f(x)] \geq f[E(x)]$

   When will the equality sign hold good in the above equation ?

2. a) State and prove the Borel zero - one law.

   b) Define conditional expectation. What is linearity property for conditional expectation ? Starting from the definition, obtain the Bayes formula for conditional probabilities.

Unit - 2

3. a) Show that the distribution function F of a random variable and its characterstic function determine each other.

   b) Let $S_N = X_1 + X_2 + \dots X_N$ where $X_1, X_2 \dots$ are i.i.d random variables with characterstic function $\phi(t)$ and N is a Poisson variate with mean λ. Find (a) the characterstic function of S_N (b) Mean and Variance of S_N.

4. a) Define a bivariate characterstic function. Obtain the characterstic function of the Cauchy distribution and show that by properly norming one can get the probability density function of the Laplace distribution.

   b) If ϕ_i, $i = 1, 2, \dots n$ are characteristic functions, examine whether $\prod_{i=1}^{n} \phi_i$ is a characterstic function. Is the converse true ? Also state a set of necessary and sufficient condition for a function $\phi(.)$ on R to be a characteristic function.

5. a) Define convergence in probability and convergence in distribution.

   b) Show that if $X_n \xrightarrow{P} X$ then $X_n \xrightarrow{L} X$.

   c) If $X_n \xrightarrow{L} X$ and $Y_n \rightarrow c$, where c is a constant, show that $X_n Y_n \xrightarrow{L} cx$.

Turn Over

6. a) State the continuity theorem on characteristic functions and discuss its applications.

b) Prove that $\Sigma(X_n - E(X_n))$ converges a.s. provided $\Sigma V(X_n) < \infty$

c) If $\{X_n\}$ is a sequence of random variables such that $X_n \xrightarrow{r} X$, show that this implies $E|X_n|^r \to E|X|^r$

7. State and prove the Kolmogrov's strong law of large numbers for the i.i.d case.

8. a) Examine whether the Kolmogrov's generalised weak law of large numbers holds for the sequence $\{X_n\}$ of i.i.d random variables with common p.d.f

$$f(x) = \frac{c}{x^2 \log x}; \ x \geq c$$

b) Let $\{X_n\}$ be a sequence of random variables which have uniformly bounded variances. If $Cov(X_i, X_j) \to 0$ as $|i-j| \to \infty$, prove that the W.L.L.N holds for $\{X_n\}$

c) State law of iterated logarithm and discuss its implications.

Unit - 3

9. a) Define infinitely divisible distribution and show that an infinitely divisible characterstic function will have no real zeros.

b) Examine whether the Gamma distribution is infinitely divisible.

c) Show that the distribution of the sums of independent random variables having infinitely divisible distributions functions is itself infinitely divisible.

d) Illustrate by an example a situation where the central limit theorem does not hold.

10. a) Show that the Lindberg form of central limit theorem implies the Lyapounov form.

b) State and prove the Lindberg - Feller form of the central limit theorem.
