M 18103

Reg. No. : ....

Name : ....

Second Year M.Sc. (S.D.E.) Degree Examination, November 2010
(2006 and Earlier)
MATHEMATICS (Elective – II)
Paper – XVII : Operations Research

Time : 3 Hours Max. Marks : 75

Instructions : *1) Answer two questions from Unit – **I** and three questions from Unit – **II**.*
*2) All questions carry **equal** marks.*

UNIT – I

I. a) Define a convex function. Let f(X) be a convex differentiable function defined in a convex domain $K \subseteq E_n$. Then prove that $f(X_0)$, $X_0 \in K$ is a global minimum if and only if $(X - X_0)' \nabla f(X_0) \geq 0$ for all X in K.

b) Prove that $f(X) = \|X\|$, $X \in E_n$ is a convex function.

c) Explain Mathematical programming in its general form.

II. a) Define basic solution of an LPP. Prove that a basic solution of the LPP is a vertex of the convex set of feasible solutions.

b) What are simplex multipliers ? What are their uses ?

III. a) What is dual of an LPP. If f(X) and ϕ (Y) are the primal and dual objective functions show that min f(X) $\geq$ max ϕ (Y).

b) Solve using dual simplex method.

Minimize $z = 2x_1 + 3x_2$

Subject to $2x_1 + 3x_2 \leq 30,$

$x_1 + 2x_2 \geq 10,$

$x_1 \geq 0, x_2 \geq 0$

P.T.O.

IV. a) Write an explanatory note on caterer problem.

b) Solve the integer programming problem using branch and bound method.

Maximize $z = x_1 + 2x_2$

Subject to $5x_1 + 7x_2 \leq 21$,

$-x_1 + 3x_2 \leq 8$.

x_1, x_2 non-negative integers.

UNIT – II

V. a) Describe a non-linear programming problem. Prove that a sufficient condition that X_0 is a minimal point of f(X) subject to the constraints. $G(X) \leq 0$ is that the function $F(X, Y) = f(X) + Y' G(X)$ has a saddle point (X_0, Y_0), $Y_0 \geq 0$.

b) Minimize $f = (x_1 + 1)(x_2 - 2)$ over the region $0 \leq x_1 \leq 2, 0 \leq x_2 \leq 1$, by writting Kuhn-Tucker conditions and obtaining the saddle point.

VI. a) Derive Kuhn-Tucker conditions for a quadratic programming problem.

b) Solve : Minimize $f = -x_1 - x_2 - x_3 + \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2\right)$

Subject to $x_1 + x_2 + x_3 - 1 \leq 0$

$4x_1 + 2x_2 - 7/3 \leq 0$

$x_1, x_2, x_3 \geq 0$

VII. a) Discuss the general method of solving a geometric programming problem.

b) Minimize $f(x) = \dfrac{40}{x_1 x_2 x_3} + 40x_2 x_3$

Subject to $g_1(x) = \frac{1}{2} x_1 x_3 + \frac{1}{4} x_1 x_2 = 1$

$x_j > 0, j = 1, 2, 3.$

VIII. a) Using dynamic programming techniques show that $\sum_{i=1}^{n} p_i \log p_i$

Subject to the constrain $\sum_{i=1}^{n} p_i = 1, p_i > 0$ is minimum when all pi's are equal to 1/n.

b) Maximize $v_1^2 + v_2^2 + v_3^2$ subject to $u_1 u_2 u_3 \leq 6$ and u_1, u_2 are u_3 are position integers.

IX. a) Write an explanatory note on backward and forward recursion techniques in dynamic programming.

b) Solve graphically the game with pay off matrix $\begin{bmatrix} 1 & 2 & 3 \\ 5 & 1 & -2 \end{bmatrix}$

X. a) Prove that for an $m \times n$ matrix game both $\max_X \min_Y E(X, Y)$ and $\min_Y \max_X E(X, Y)$ exist and are equal.

b) Using LP techinques solve the game whose pay-off matrix is given by

$$\begin{bmatrix} -2 & -4 & 0 \\ 0 & 2 & -6 \\ 3 & -1 & -5 \end{bmatrix}$$