M 15145

Reg. No. : ....

Name : ....

Third Semester M.Sc. Degree Examination, November 2008
STATISTICS
Paper – 3.2 : Multivariate Analysis

Time : 3 Hours | Max. Marks : 70

Instructions : *1) Answer **any** 5 questions, without omitting **any** Unit.*
*2) **All** questions carry **equal** marks.*

UNIT – I

I. a) Derive the probability density function of multivariate normal distribution.

b) Show that when $X \sim N_2(\mu, \Sigma)$, the conditional distribution will have expected value in the form

$$E(X_1|X_2=x_2) = \mu_1 + \rho\frac{\sigma_1}{\sigma_2}(x_2-\mu_2) \text{ with residual variance}$$

$$V(X_1|X_2= x_2) = \sigma_1^2(1-\rho^2).$$

II. a) Obtain the characteristic function of a multivariate normal distribution and show that linear functions of normally distributed random variables are normally distributed.

b) State and prove Cochran's theorem.

UNIT – II

III. a) Show that the sample mean vertal and sample covariance matrix of a random sample from a multivariate normal distribution are independent. Obtain unbiased estimators for μ and Σ.

b) Let $X \sim N_p(\mu, \Sigma)$, Σ is positive definite. Show that $Q = (X-\mu)' \Sigma^{-1} (X-\mu)$ is distributed as chi-square variate with p degrees of freedom.

P.T.O.

IV. a) Give the definition of Wishart distribution as a generalization of chi-square. Derive the characteristic function of Wishart distribution and hence prove its additive property.

b) A Wishart matrix V is partitioned into $\begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}$. Find the distribution of

$V_{11.2} = V_{11} - V_{12} V_{22}^{-1} V_{21}$.

UNIT – III

V. a) Obtain the likelihood ratio criterion for testing independence of sets of variates when $X \sim N_p(\mu, \Sigma)$.

b) Derive the likelihood ratio test for testing the equality of mean vectors of two multivariate normal population having the same covariance matrix (unknown).

VI. a) Derive a test for $\Sigma_1 = \Sigma_2$ with two samples of respective sizes n_1 and n_2 from $N(\mu_1, \Sigma_1)$ and $N(\mu_2, \Sigma_2)$.

b) $X \sim N_p(0, \Sigma)$. Test the hypothesis that $\Sigma = \sigma^2 I$ where σ^2 is known.

UNIT – IV

VII. a) Define partial correlation coefficient. Derive the distribution of the sample partial correlation coefficient in the null case.

b) Define hotelling T^2 statistic. What are the uses of T^2 ? Explain.

VIII. a) Write down Mahalanobis D^2 statistic and explain how it can be used for testing the equality of mean vectors of two p-variate normal populations with same unknown dispersion matrix.

b) What is multivariate Fisher–Behren's problem ? Explain.

UNIT – V

IX. a) Describe the problem of discriminating between the two populations π_1 and π_2 on the basis of measurements on a p-component vector of variables X.

b) Define discriminant function. Derive Fisher's linear discriminant function.

X. a) Define principal components. Obtain the first principal component when the dispersion matrix Σ is of the form $\sigma^2 \begin{bmatrix} 1 & \rho & \rho \\ \rho & 1 & \rho \\ \rho & \rho & 1 \end{bmatrix}$.

b) Explain how the estimation of canonical correlations and variates are made by stating the assumptions made.
