Reg. No. : ....

Name : ....

M 13834

Third Semester M.Sc. Degree Examination, November 2007

STATISTICS

Paper – 3.2 : Multivariate Analysis

Time: 3 Hours   Max. Marks: 70

Instructions : *1) Answer **any 5** questions without **omitting any** Unit.*

*2) All questions carry **equal** marks.*

UNIT – 1

I. a) If X is $N(0, I_p)$, obtain a necessary and sufficient condition for the independence of the quadratic forms $X'AX$ and $X'BX$.

b) Obtain the characteristic function of $N_p(\mu, \Sigma)$. Hence or otherwise, deduce the distribution of $Y = CX$ where $X \sim N_p(\mu, \Sigma)$ and C is a $(p \times p)$ matrix of rank p.

II. a) Show that X follow $N_p(\mu, \Sigma)$ if and only if every linear combination $l'x$ has univariate normal distribution.

b) When will you say that a multivariate normal distribution is singular ? If (X_1, X_2) has a bivariate normal distribution, show that X_1 and X_2 are independent if and only if the correlation coefficient $\rho = 0$. Also, write down the probability density function of any one non-normal distribution of your choice and obtain the marginals.

UNIT – 2

III. a) For a multivariate normal population, show that the sample mean $\overline{X}$ and the sample dispersion matrix S are independent.

b) Define a Wishart distribution. Show that it is the multivariate analogue of Chi-square distribution. Explain how one can obtain the distribution of rectangular co-ordinates from the distribution of the Wishart matrix.

IV. a) Obtain the maximum likelihood estimators of μ and Σ using a random sample from $Np(\mu, \Sigma)$. State whether they are (a) unbiased (b) consistent.

b) What do you understand by generalised variance? Also state and prove the reproductive property of the Wishart distribution.

P.T.O.

UNIT – 3

V. Using likelihood ratio principle, derive a test for testing the equality of co-variance matrices of two p-variate normal populations with known mean vectors. Also derive the test for testing the hypothesis that a p-variate normal population has a specified mean vector and co-variance matrix.

VI. a) X follow $N_p(\mu, \Sigma)$ and X is partitioned as $X = (X^{(1)}, X^{(2)})'$. Test the hypothesis that $X^{(1)}$ and $X^{(2)}$ are independent.

b) Write short notes on :

i) Asymptotic distribution of the likelihood ratio criterion and

ii) Multivariate Fisher-Behren problem.

UNIT – 4

VII. a) Derive the sampling distribution of the partial correlation coefficient in the null case.

b) Define Hotelling's T^2 and Mahalanobis D^2 statistics. What are its uses ? Write down the relation between T^2 and D^2.

VIII. a) Define the partial and multiple correlation coefficients for a p variate distribution. Obtain the distribution of the sample multiple correlation coefficient.

b) Describe any one test procedure using the T^2 statistic. State the invariant property of the T^2 test.

UNIT – 5

IX. a) Define Fisher's discriminent function and explain how you could estimate it for classifying individuals into one of two normal populations with unknown parameters. Point out its relation with D^2 statistic.

b) What are principal components ? Show that they are uncorrelated. Explain the procedure to construct principal components.

X. Write short notes on :

1) Canonical variates and canonical correlation
2) Factor analysis
3) Classification problem
4) Sphercity test.

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