Reg. No. : ....

Name : ....

M 12517

Third Semester M.Sc. Degree Examination, November 2006
STATISTICS
Paper 3.2 : Multivariate Analysis

Time: 3 Hours

Max. Marks: 70

Instructions: *1) Answer any 5 questions without omitting any Unit.*

2) All questions carry equal marks.

UNIT – I

I. a) A random vector (X, Y) has pdf $f(x, y) = \frac{1}{2}\sin(x+y), 0 \le x \le \frac{\pi}{2}, 0 \le y \le \frac{\pi}{2}$.
Determine the distribution function and covariance matrix of (X, Y).

b) Show that a P-variate random vector $\underset{\sim}{X}$ has multivariate normal distribution if and only if every linear combination of $\underset{\sim}{X}$ is univariate normal.

II. a) The random vector (X, Y) has bivariate normal distribution. Show that X + Y and X – Y are independent if and only if $\sigma_x^2 = \sigma_y^2$. Use this result to show that $\overline{X}$ and S^2 are independent if they are calculated based on a random sample of size two from a univariate normal distribution.

b) Define singular normal distribution. Describe how this is different from non-singular normal distribution.

UNIT – II

III. a) Write down the MLEs of the parameters of a non-singular normal distribution. Check whether they are :

i) Independent

ii) Unbiased estimators of the parameters

iii) Consistant estimators of the parameters.

P.T.O.

b) Define Wishart distribution. Enumerate its properties. Deduce Chi-square distribution as its particular case.

IV. a) What is sample generalized variance ? Derive its distribution.

b) If $A \sim W(n, \Sigma)$, derive

i) $E(A)$ ii) $E|A|$ and   iii) $E(A^{-1})$

UNIT – III

V. a) Given $X \sim N_p(\mu, \Sigma)$, where Σ is known. Explain how you test $H_0 : \underset{\sim}{\mu} = \underset{\sim}{\mu_0}$, a given vector. Also explain how you construct confidence region for mean vector μ.

b) In $N_p(\mu, \Sigma)$ both μ and Σ are unknown, derive the likelihood ratio criterion for testing $H_0 : \Sigma = \sigma^2 I$.
What is the approximate test here ?

VI. Let $\underset{\sim}{X}^{(i)}$ be distributed like $N_p(\mu^{(i)}, \Sigma^{(i)})$, i = 1, 2. Derive the likelihood ratio criterions, say $\lambda_1, \lambda_2, \lambda$ respectively, for testing the following hypotheses.

$H_1 : \underset{\sim}{\mu}^{(1)} = \underset{\sim}{\mu}^{(2)}$, given $\Sigma^{(1)} = \Sigma^{(2)}$

$H_2 : \Sigma^{(1)} = \Sigma^{(2)}$

$H : \mu^{(1)} = \mu^{(2)}$ and $\Sigma^{(1)} = \Sigma^{(2)}$.

Verify that $\lambda = \lambda_1 \lambda_2$.

UNIT – IV

VII. a) Define Hotelling's T^2 – statistic. Show that T^2 is the statistic to be used to test $H_0 : \underset{\sim}{\mu} = \underset{\sim}{\mu_0}$, a given vector in $N(\mu, \Sigma)$ when Σ is unknown.

b) State and prove the invariance property of T^2 – statistic.

VIII. a) Derive the sampling distribution of simple correlation coefficient in the null case.

b) Define Mahalnobis D^2– statistic. Describe its applications in testing problems.

UNIT – V

IX. a) Explain the likelihood ratio method of classification of individuals belonging to two normal populations.

b) Obtain the distribution of the criterion you use in (a).

X. a) Explain the procedures to construct principal components when the population dispersion matrix is known. What are the percentage of variation covered by each principal component ?

b) What is factor analysis ? Describe the principle component solution method for the estimation of factors.
