M 11187

Third Semester M.Sc. Degree Examination, November 2005

STATISTICS

Paper 3.2 : Multivariate Analysis

(2004 Admn.)

Time : 3 Hours   Max. Marks : 70

Instructions : 1) Answer any 5 questions without omitting any Unit.
2) All question carries equal marks.

UNIT – I

I. a) Define multivariate non-singular normal distribution. Identify its parameters.

b) Show that a random vector $\underset{\sim}{X}$ has multivariate normal distribution if and only if every linear combination of $\underset{\sim}{X}$ is univariate normal.

II. a) If $\underset{\sim}{X}$ is distributed according to $N_p(\mu, \Sigma)$, show that $Z = D\underset{\sim}{X}$ is distributed according to $N_q(D\mu, D\Sigma D')$, where D is a $q \times p$ matrix of rank q, $(q \le p)$.

b) Let $\underset{\sim}{X} \sim N_p(\mu, \Sigma)$ and $\underset{\sim}{X}$ is partitioned as $X = \left(X^{(1)}_{q\times 1}, X^{(2)}_{(p-q)\times 1}\right)$. Write down the conditional distribution of $X^{(1)}$ given $X^{(2)} = X^{(2)}$. Describe how this conditional distribution helps you to determine the following :

i) The regression function of $X^{(1)}$ on $X^{(2)} = x^{(2)}$.

ii) Partial correlation between $X_i, X_j \in X^{(1)}$ given $X^{(2)} = x^{(2)}$.

iii) Multiple correlation between $X_i \in X^{(1)}$ and $X^{(2)}$

UNIT – II

III. a) Determine the maximum likelihood estimators of μ and Σ in $N_p(\mu, \Sigma)$. Check whether they are unbiased estimators of the parameters.

b) What is generalized variance. Derive the distribution of sample generalized variance.

IV. a) Define Wishart distribution. Explain why this is a generalization of chi-square distribution. Derive the characteristic function of Wishart distribution.

b) A positive definite matrix D be $W_p(N, \Sigma)$ distributed and H be any $m \times p$ orthogonal matrix of rank m. Prove that HDH' is $W_m(N, H\Sigma H')$ distributed.

P.T.O.

UNIT – III

V. a) Let $\underset{\sim}{X} \sim N_p\left((\mu, \Sigma\right)$ and $\underset{\sim}{X}$ is partitioned as $\underset{\sim}{X} = \left(X^{(1)}, X^{(2)}\right)$. Derive the likelihood ratio criterion for testing $H_0 : X^{(1)}$ and $X^{(2)}$ are independent.

b) What is the asymptotic test for the testing problems above ?

VI. a) Let $N_p\left(\mu^{(g)}, \Sigma_g\right)$, g = 1, 2, ..., q be q multivariate normal populations. Derive the λ-criterion for testing $H_0 : \Sigma_1 = \Sigma_2 = ... = \Sigma_g$.

b) Describe the asymptotic test for the above test.

UNIT – IV

VII. a) Describe multivariate Fisher-Behren's problem. How do you solve it ?

b) Define Mahalnobis D^2 – statistic. Describe how you use this to test the equality of mean vectors of two multivariate normal populations when they have equal covariance matrix.

VIII. a) Define partial correlation coefficient for a p-variate normal distribution. Derive the sampling distribution of partial correlation coefficient in the null case.

b) Determine the sampling distribution of sample correlation coefficient in the case of a bivariate normal population with zero correlation coefficient.

UNIT – V

IX. a) Describe the classification problem into one of two known multivariate normal populations with equal dispersion matrix.

b) What are principal components. Illustrate how you construct principal components of $\underset{\sim}{X} \sim N_P\left(\mu, \Sigma\right)$ when Σ is known.

X. a) Give a comparative discussion of principal component analysis and factor analysis.

b) Discuss the problems of estimating canonical correlations and variates in normal populations.
