M 24423

Reg. No. : ....

Name : ....

I Semester M.A./M.Sc./M.Com./M.Sc. Comp. Science Degree (Reg./Sup./Imp.) Examination, November 2013
STATISTICS
Paper – 1.3 : Linear Algebra and Numerical Methods

Time: 3 Hours Max. Marks : 60

Instructions : *1)* ***All*** *questions carry* ***equal*** *marks.*
2) Answer ***any five*** *questions without omitting* ***any*** *Unit.*

UNIT – 1

1. a) Define subspace of a vector space. State and prove a necessary and sufficient condition for a subset V_1 of a vector space V to be a subspace. 6

   b) Show that any set of non-zero orthogonal vectors is linearly independent. 6

2. a) Define an idompotent matrix. Let A be an idompotent matrix of order n. Then prove or disprove Rank (A) + Rank (I – A) = n, where I is the identity matrix. 6

   b) If V is a finite dimensional inner product space show that V has an orthonormal basis. 6

UNIT – 2

3. a) Define a quadratic form. Show that every quadratic form can be reduced to a form containing square form only by a nonsingular linear transformation. 6

   b) Examine whether the quadratic form

   $$Q = x_1^2 + x_1x_2 - x_1x_3 + x_2^2 + 4x_2x_3 + x_3^2$$

   is positive definite. 6

P.T.O.

4. a) Define eigen values and eigen vectors of a square matrix. Find the eigen values and eigen vectors of

$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & 0 \\ 2 & 2 & 3 \end{pmatrix}$$ 6

b) Prove that eigen vectors corresponding to two distinct eigen values of a Hermitian matrix are orthogonal. 6

UNIT – 3

5. a) Show that a consistent system A X = b has a unique solution iff A is a full column rank. 6

b) If a and b are real numbers, not both zero, show that the three lines

$ax + by - a - b = 0$

$bx - (a + b)y + a = 0$

$(a + b)x - ay - b = 0$

in $\mathbb{R}^2$ are distinct and concurrent. 6

6. a) Explain any one method of matrix inversion by a matrix reduction technique. 6

b) Let λ be an eigen value of a $n \times n$ matrix A. Then show that the geometric multiplicity of λ is less than or equal to the algebraic multiplicity. 6

UNIT – 4

7. a) Define generalized inverse of a matrix. Explain any one method of computing g-inverse. 6

b) Let G be a g-inverse of A. Then show that the system AX = b is consistent iff A Gb = b. 6

8. a) Define Moore-Penrose inverse. Show that it is unique. **6**

b) Find Moore-Penrose inverse of **6**

$$A = \begin{bmatrix} 1 & 0 & 3 \\ -1 & 2 & 4 \end{bmatrix}$$

UNIT – 5

9. a) Explain Graffe's root squaring method of solving a an algebraic equation. **6**

b) Find a root of the equation $x^3 - 3x - 5 = 0$ by using Newton-Raphson method. **6**

10. a) Discuss method of finding a numerical solution of a differential equation by Taylor's series. **6**

b) Given the differential equation

$$\frac{dy}{dx} = \frac{x^2}{y^2 + 1} \text{ with } y(0) = 0$$

Use Picard's method to obtain y for x = .25, .5 correct to three decimal places. **6**
