



Reg. No. :

Name :

I Semester M.A./M.Sc./M.Com. Degree (Reg./Sup./Imp.)

Examination, November 2012

STATISTICS

Paper – 1.3 : Linear Algebra and Numerical Methods

Time: 3 Hours

Max. Marks: 60

Instructions : 1) **All** questions carry **equal** marks.2) Answer **any five** questions without omitting **any** Unit.

UNIT – 1

1. a) Define basis and dimension of a vector space. Show that $B = \{1 + x, x + x^2, 1 + x^2\}$ is a basis for p_2 , the set of all polynomials of degree less than or equal to two. 6
- b) Let V be a vector space with dimension n . Show that any linearly independent set in V can be extended to a basis of V . 6
2. a) Define rank of a matrix. Let $r(A)$ be the rank of a matrix. Show that $r(AB) \leq \text{Min}[r(A), r(B)]$. 6
- b) Describe the method of finding inverse of a matrix by forming a partition of A . 6

UNIT – 2

3. a) Illustrate the different types of quadratic form and state a condition characterizing each type of the quadratic form. 6
- b) Classify the quadratic form $9x_1^2 + 4x_2^2 + 4x_3^2 + 8x_1x_2 + 12x_1x_3 + 12x_1x_2$. 6
4. a) Find eigen values and eigen vectors of the matrix :

$$A \begin{pmatrix} 0 & 1 & -1 \\ 1 & 1 & 1 \\ 1 & 2 & 0 \end{pmatrix}$$

- b) Prove that characteristic roots of real symmetric matrix are real. Also show that characteristic vectors also can be chosen real. 6

P.T.O.



UNIT - 3

5. a) Show that the only possible eigen values of an idempotent matrix are zero and one. 6
- b) Determine the geometric and algebraic multiplicities of eigen values of the matrix 6

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 1 & 2 \\ 0 & 2 & 1 \end{pmatrix}$$

6. a) Solve the system of equations 6
- $$2x + 6y + z = -1$$
- $$3x + 9y + 2z = -1$$
- $$-y + 3z = 4$$
- by any of the reduction technique. 6
- b) State and prove spectral decomposition theorem for a real symmetric matrix. 6

UNIT - 4

7. a) Define generalized inverse of a matrix. Show that every matrix has a g-inverse. 6
- b) For any two matrices A and G show that the following statements are equivalent. 6
- i) G is a g-inverse of A.
 - ii) AG is idempotent and $\text{rank}(AG) = \text{rank}(A)$.
8. a) If G_1 and G_2 are two g-nverses of A, show that $\alpha G_1 + (1 - \alpha) G_2$ is also a g-inverse of A for all $\alpha \in F$. 6
- b) Find the g-inverse of the matrix 6

$$A = \begin{pmatrix} 1 & 2 & 4 \\ -1 & 0 & 3 \\ 2 & 4 & 8 \end{pmatrix}$$



UNIT - 5

9. a) Explain Regula-falsi method of solving a transcendental equation. 6
b) Find the root of the equation
 $x^3 - 3x - 5 = 0$
by Graffe's root squaring method. 6
10. a) Given that $\frac{dy}{dx} = 1 + y^2$, where $y(0) = 0$. Use the fourth-order Runge-Kutta formula to find $y(0.2)$. 6
b) Discuss Euler's method for solving a differential equation numerically. 6