M 20484

Reg. No. : ....

Name : ....

I Semester M.A./M.Sc./M.Com. Degree (Reg./Sup./Imp.) Examination, November 2011
STATISTICS
Paper – 1.3 : Linear Algebra and Numerical Methods

Time: 3 Hours Max. Marks : 60

Instructions : *1)* ***All*** *questions carry* ***equal*** *marks.*
2) Answer ***any five*** *questions without omitting* ***any*** *Unit.*

UNIT – 1

1. a) When do you say that a set of vectors $a_1, a_2, \ldots a_n$ of a vector space is linearly independent ? Suppose V is a vector space of dimension n, show that any subset V which contains more than n elements must be linearly dependent. **6**

b) Determine whether or not each of the following set is a basis set of the vector space R^3. **6**

i) {(5, 3, 7), (1, 3, 6), (0, 3, 1)}

ii) {(1, 2, 6), (– 1, 3, 4), (–1, – 4, – 2)}.

2. a) If A and B are two matrices of same order, then prove or disprove : **6**

Rank (A + B) = Rank (A) + Rank (B).

b) Explain Gram-Schmidt orthogonalization process. **6**

UNIT – 2

3. a) Show that a real symmetric matrix is non negative definite if and only if all the principal minors of A are nonnegative. **6**

b) Let A be positive definite and $M = \begin{bmatrix} A & b \\ b^T & d \end{bmatrix}$. Show that M is positive definite, positive semi definite or indefinite according as $d - b^T A^{-1} b$ is positive, zero or negative. **6**

P.T.O.

4. a) Define eigen values and eigen vectors of a square matrix. Let $\alpha_1, \alpha_2, ...\alpha_k$ be distinct eigen values of A and let $X_1, X_2, ... X_k$ be corresponding eigen vectors. Then $X_1, X_2, ... X_k$ are linearly independent. 6

b) For any eigen value α of A, show that the algebraic multiplicity of α with respect to A is not less than geometric multiplicity of α with respect to A. 6

UNIT – 3

5. a) Prove that two characteristic vectors corresponding to two distinct characteristic roots of a Hermitian matrix are orthogonal. 6

b) Prove that any real symmetric matrix is orthogonally similar to a diagonal matrix. 6

6. a) Let $A = XX^T$ where $X = (1, 2, 1, 4)^T$ obtain a spectral decomposition of A. 6

b) Let A be a real symmetric matrix

i) $A^K = I$ for some +ve integer K, then show that $A^2 = I$.

ii) If eigen values of A are all positive and $A^K = I$ for some positive integer K, show that A = I. 6

UNIT – 4

7. a) Let A be a symmetric matrix and C be a positive definite matrix. The characteristic roots of $| A - \lambda \subset | = 0$ are such that $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_m$. Show that

$$\sup_x \frac{X'AX}{X' \subset X} = \lambda_1 \text{ and } \inf_x \frac{X'AX}{X' \subset X} = \lambda_m$$ 6

b) Let A be an m × n matrix and let G be a g-inverse of A. Then show that A general solution of AX = 0 is (I – GA)z where z is an arbitrary vector of F^n. 6

8. a) If A is a matrix of full column rank show that G is a g-inverse of A if and only if G is a left inverse of A. 6

b) Find the g – inverse of

$$A = \begin{bmatrix} 3 & 4 & 7 \\ 2 & 1 & 5 \\ 1 & 3 & 2 \end{bmatrix}$$ 6

UNIT – 5

9. a) Find a real root of the equation $x^3 - x - 1$ by bisection method. 6

b) Discuss the Newton-Raphson method for solving an equation. Also discuss the rate of convergence of the method. 6

10. a) Discuss Picard's iterative method of solving a differential equation. 6

b) Solve by the modified Euler method, the problem 6

$$\frac{dy}{dx} = \frac{1}{2} - x + 2y, \ y(0) = 1$$

Take h = .1 and compute y(.1), y(.2), y(.3) and y(.4).
