M 16933

Reg. No. : .....

Name : .....

First Semester M.Sc. Degree Examination, November 2009
STATISTICS
Paper – 1.3 : Linear Algebra and Numerical Methods

Time: 3 Hours

Max. Marks: 60

Instructions *:1)* ***All*** *questions carry* ***equal*** *marks.*

2) Answer ***any five*** *questions without omitting* ***any*** *Unit.*

UNIT – I

1. i) Define linear vector space, give an example. Prove that every linear space (of m × n matrices) has a basis.

   ii) Show that the vectors (1, 2, 1), $V_2 = (1, -1, 1)$, $V_3 = (3, 3, 3)$ form a linearly dependent set. Also express one of these as a linear combination of the others.

OR

2. i) Explain Gram-Schmidt orthogonalization process.

   ii) Define rank of a matrix. Show that the rank of the product of two matrices cannot exceed the rank of either factor.

UNIT – II

3. i) Define a positive definite quadratic form. State and prove a necessary and sufficient condition that a real quadratic form is positive definite.

   ii) Show that the definiteness of a quadratic form is invariant under non-singular linear transformation.

OR

4. i) Define characteristic root and characteristic vector. If P_i and P_j are characteristic vectors corresponding to two distinct characteristic roots λ_i and λ_j prove that P_i and P_j are orthogonal to each other.

   ii) If A is a real symmetric matrix, prove that all the latent roots are real and the latent vectors can be chosen to be real.

P.T.O.

UNIT – III

5. i) Prove that any two characteristic vectors corresponding to two distinct characteristic roots of a unitary matrix are orthogonal.

   ii) Distinguish between algebraic and geometric multiplicity of an eigen value.

   Find the algebraic and geometric multiplicity of the matrix $\begin{bmatrix} 0 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 1 & 4 \end{bmatrix}$.

OR

6. i) Reduce to the diagonal form $-y^2 + 2yz - 2xy$. Also give the appropriate linear transformation.

   ii) Explain the spectral decomposition of a matrix. How will you obtain the inverse of a matrix using spectral decomposition ?

UNIT – IV

7. i) Let A be a symmetric matrix and C be a positive definite matrix. The characteristic roots of $|A - \lambda C| = 0$ are such that $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_m$.

   Show that $\sup_X \frac{X'AX}{X'CX} = \lambda_1$ and $\inf_X \frac{X'AX}{X'CX} = \lambda_m$.

   ii) Define generalized inverse of a matrix. Explain any one method of computing g-inverse.

OR

8. i) Prove that $\overline{A}$ is a g-inverse if and only if $A\overline{A}A = A$.

   ii) Find the g-inverse of the matrix $\begin{bmatrix} 4 & 1 & 2 & 0 \\ 1 & 1 & 5 & 15 \\ 3 & 1 & 3 & 5 \end{bmatrix}$.

UNIT – V

9. i) Explain bisection method for solving transcendental equations.

   ii) Explain the procedure of Newton-Raphson method. Comment on the number of iterations required for converging to a root.

OR

10. i) Explain the procedure of finding an initial approximation to the transcendental equations. Give an example.

    ii) Describe Euler method to get the numerical solution of a differential equation.
