M 15469

Reg. No. : ....

Name : ....

First Semester M.Sc. Degree Examination, December 2008
STATISTICS
Paper – 1.3 : Linear Algebra and Numerical Methods

Time : 3 Hours Max. Marks : 60

Instructions :1) All questions carry equal marks.
2) Answer any five questions without omitting a Unit.

UNIT – I

1. i) Define linear independence and dependence of vectors. Show that a system of non-null mutually orthogonal vectors are linearly independent.

   ii) Check for linear dependence or independence in the following set of vectors.
   $V_1 = (2, 0, 1, -1), V_2 = (3, 0, -1, 2), V_3 = (5, 0, 0, 1)$.

OR

2. i) Define rank of a matrix. Let A be an idempotent matrix of order m. Show that rank (A) = trace (A) and rank (A) + rank (I – A) = m.

   ii) Define :
   a) Vector subspace
   b) Spanning set of a vector subspace
   c) Dimension of a vector subspace.

UNIT – II

3. i) State and prove a necessary and sufficient condition that a real quadratic form $X'AX$ is positive definite.

   ii) Classify the quadratic form $x_1^2 + 4x_2^2 + x_3^2 - 4x_2x_3 + 2x_3x_1 - 4x_1x_2$ as definite, semi definite or indefinite.

OR

P.T.O.

4. i) State and prove Cayley-Hamiltan theorem.

   ii) Show that the two matrices A, $P^{-1}AP$ have the same characteristic roots.

UNIT – III

5. i) Prove that the modulus of each characteristic root of a unitary matrix is unity.

   ii) Define the terms algebraic multiplicity and geometric multiplicity. Prove that geometric multiplicity of a characteristic root cannot exceed algebraic multiplicity of the same.

OR

6. i) If A, be a real symmetric matrix, then prove that there exists an orthogonal matrix P such that $P^1AP = P^{-1}AP$ is a diagonal matrix with real elements.

   ii) Find a real non-singular linear transformation which reduces simultaneously

   $3x^2 + 5y^2 + 5z^2 + 2yz + 6zx - 2xy,$

   $5y^2 + 12y^2 + 8yz + 4xz,$

   to the sum of squares form.

UNIT – IV

7. i) Define g-inverse of a matrix and show that it is not unique.

   ii) Obtain the g-inverse of the matrix $A=\begin{bmatrix} 4 & 1 & 2 \\ 1 & 1 & 5 \\ 3 & 1 & 3 \end{bmatrix}$.

   Show that $A\bar{A}A = A$.

OR

8. i) Show that $\bar{A}$ is a g-inverse $\Leftrightarrow \bar{A}A$ is idempotent and $\rho(\bar{A}A) = \rho(A)$.

   ii) Construct Moore-Penrose g-inverse of $A = \begin{bmatrix} 1 & -1 & 2 \\ 1 & -2 & 3 \end{bmatrix}$ and hence solve the equation $AX = b$ where $b' = (2, 2)$.

UNIT – V

9. i) What are transcendal equations ? How will you obtain an initial approximation of it ? Given the following equation $x^4 - x - 10 = 0$, determine the initial approximation.

ii) Describe Newton Raphson method. Show that the initial approximation x_0 for finding $1/N$, where N is a positive integer, by Newton - Raphson method must satisfy $0 < x_0 < 2/N$, for convergence.

OR

10. i) Explain Graeffe's Root squaring method.

ii) Describe the Runga-Kutta method.