Reg. No. : ....

Name : ....

M 13935

First Semester M.Sc. Degree Examination, November 2007
STATISTICS
Paper – 1.3 : Linear Algebra and Numerical Methods

Time : 3 Hours Max. Marks : 60

***Instructions :** 1) All questions carry equal marks.*

2) Answer any five questions without omitting a Unit.

UNIT – I

1. i) Define linearly dependent and linearly independent set of vectors. Show that every linearly independent set of vectors $(\xi_1, \xi_2, \ldots, \xi_n)$ can be extended so as to constitute a basis of V_n.

   ii) Prove that every subspace, S, of V_n has a basis.

OR

2. i) Construct an orthogonal basis of a vector space. Explain the Gram-Schmidth orthogonalization process.

   ii) For any m × m matrices A and B, prove that rank (A + B) ≤ rank (A) + rank (B).

UNIT – II

3. i) Define 'quadratic form'. State and prove the necessary and sufficient condition for a quadratic form to be positive definite.

   ii) Examine the definiteness of the quadratic form

$$5x_1^2 + 26x_2^2 + 10x_2^2 + 4x_2x_3 + 14x_3x_1 + 6x_1x_2$$

OR

4. i) Define 'Gram matrices'. Prove that every positive definite or semi definite matrix can be represented as a Gram matrix.

   ii) Show that the two matrices A, $P^{-1}AP$ have the same characteristic roots.

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UNIT – III

5. i) Prove that any two characteristic vectors corresponding to two distinct characteristic roots of a real symmetric matrix are orthogonal.

ii) If $X'AX$ be any real quadratic form of rank r, then prove that there exists a real orthogonal transformation $X = PY$ which transforms the form to $\lambda_1 y_1^2 + \lambda_2 y_2^2 + ... + \lambda_r y_r^2$, where $\lambda_1, \lambda_2, ..., \lambda_r$ are the, r non-zero characteristic roots of the matrix A; n – r characteristic roots of A being equal to zero.

OR

6. i) Let A be n × n matrix with n distinct characteristic roots $\lambda_1, \lambda_2, ..., \lambda_n$. Then there exists an invertible matrix P such that $P^{-1}AP = \text{diag}(\lambda_1, \lambda_2, ..., \lambda_n)$.

ii) Obtain the spectral decomposition, of the following matrix and hence find out its powers $A = \begin{bmatrix} 2 & -4 & 2 \\ -4 & 2 & -2 \\ 2 & -2 & -1 \end{bmatrix}$

UNIT – IV

7. i) Define generalised inverse and reflexive generalised inverse. Show that generalised inverse is not unique.

ii) Find the generalised inverse of $A = \begin{bmatrix} 4 & 1 & 2 & 0 \\ 1 & 1 & 5 & 15 \\ 3 & 1 & 3 & 5 \end{bmatrix}$.

OR

8. i) Prove that generalised inverse of A exist if and only if AGA = A where G is the generalised inverse of A.

ii) Show that Moore Pentrose generalised inverse is unique.

UNIT – V

9. i) Describe Newton-Raphson method and discuss its rate of convergence.

ii) Describe the Taylor series method for obtaining numerical solution of a differential equation.

OR

10. i) Explain the Graeffe's root squaring method when all the roots are real and distinct.

ii) Describe Regula-Falsi method for find the root of $f(x) = 0$.

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