Reg. No. : ....

Name : ....

M 12702

First Semester M.Sc. Degree Examination, November 2006
STATISTICS
Paper – 1.3 : Linear Algebra and Numerical Methods

Time : 3 Hours Max. Marks : 60

Instructions : *1) All questions carry equal marks.*
2) Answer any five questions without omitting a unit.

UNIT – I

1\. i) Define linear vector space, give examples. Prove that every linear space (of $m \times n$ matrices) has a basis.

ii) Show that the vectors (2, 3, –1, –1), (1, –1, –2, – 4), (3, 1, 3, –2), (6, 3, 0, –7) form a linearly dependent set. Also express one of these as a linear combination of the others.

OR

2\. i) If A, B are two n-rowed square matrices, then prove that

$\rho(AB) \geq \rho(A) + \rho(B) - n$.

ii) Show that the following sets of vectors costitute a basis of V_3 {(2, 3, 4), (0, 1, 2), (–1, 1, –1)}.

UNIT – II

3\. i) Define 'quadratic form'. Show that the form $x_1^2 + 2x_2^2 + 3x_3^2 + 2x_2x_3 - 2x_3x_1 + 2x_1x_2$ is indefinite and find two sets of values of x_1, x_2, x_3 for which the form assumes positive and negative values.

ii) If $A = (a_{ij})$ is a real positive definite matrix, show that $|A| \leq a_{11}a_{22}......a_{nn}$.

OR

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4. i) If A be any n-rowed non-zero symmetric matrix of rank r, then prove that there exists a non-singular matrix P whose elements may be any complex numbers such that $P'AP = \text{diag}(1, 1, -1, 0, 0, -0)$ where, 1 appears r times.

ii) If A and B are two square matrices, Then prove that the matrices AB and BA have the same characteristic roots.

UNIT – III

5. i) Prove that the geometric multiplicity of a characteristic root cannot exceed algebraic multiplicity of the same.

ii) If A, be a real symmetric matrix, then show that there exists an arthogonal matrix P such that $P'AP = P^{-1}AP$ is a diagonal matrix with real elements.

OR

6. i) Prove that the necessary and sufficient condition for an n-rowed matrix to be similar to a diagonal matrix is that the set of characteristic vectors of A includes a set of ,n, linearly independent vectors.

ii) Obtain the spectral decomposition the following matrix and hence find out its powers.

$$A = \begin{bmatrix} 24 & -20 & 10 \\ -20 & 24 & -10 \\ 10 & -10 & 9 \end{bmatrix}$$

UNIT – IV

7. i) State and prove the definition, existence and uniqueness of the Moore Penrose Inverse.

ii) If G is the reflexive generalised inverse of A, then show that rank (A) = rank (G).

OR

8. i) Define generalised inverse and write the importance of it. Prove that any system of consistent equation AX = Y have a solution X = GY if and only if AGA = A.

ii) Find the generalised inverse of $A = \begin{bmatrix} 1 & 2 & 5 & 2 \\ 3 & 7 & 12 & 4 \\ 0 & 1 & -3 & -2 \end{bmatrix}$.

UNIT – V

9. i) Describe the Regula-Falsi method for finding the root of f(x) = 0

ii) Explain the importance of solution of algebraic and transcendental equations.

OR

10. i) Describe bisection method for finding the root of f(x) = 0. Comment on the number of iterations required for converging to a root.

ii) Describe the Runga-Kutta method.
