First Semester M.Sc. Degree Examination, November 2005

STATISTICS

Paper 1.3 : Linear Algebra and Numerical Methods

Time: 3 Hours Max. Marks: 60

*Instructions : All questions carry **equal** marks. Answer any **five** questions without omitting a Unit.*

UNIT – I

1. Let V be vector space over the field F and let S be a set. Let $W = V^S$ be the set of all mapping from S to V. Define addition and scalar multiplication in W pointwise, that is given $f : S \to V$ and $g : S \to V$ define $f + g : S \to V$ by

   $(f + g)(s) = f(s) + g(s)$

   and

   $\alpha f : S \to V, \alpha \in F$ by

   $(\alpha f)(s) = \alpha\, f(s)$.

   Then show that W is a vector space over F.

OR

2. i) Complete the set {(2, 1, 4, 3), (2, 1, 20)} to form a basis of R^4.

   ii) Let $f : R^3 \to R^3$ be defined by $f(x_1, x_2, x_3) = (x_1, x_1+x_2, x_1+x_2+x_3)$. Show that f is a linear mapping. Also find Kernel of f.

UNIT – II

3. a) What do you mean by Jacobian Matrix ? Evaluate the Jacobian matrix of

   $y_1 = 2x_1 + x_2 - x_3 + x_4$

   $y_2 = x_1 + 3x_2 + x_3 + 2x_4$

   $y_3 = -x_1 + x_2 + x_3 - x_4$

   b) Let A be nxn matrix with n distinct eigen values $\lambda_1, \lambda_2, \ldots\ldots\lambda_n$. Then there exists an invertible matrix P such that $P^{-1}AP = \text{diag}(\lambda_1, \lambda_2, \ldots\ldots\lambda_n)$.

4\. a) Let $A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 3 \end{bmatrix}$ Find a matrix P

such that $P^{-1}AP = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$.

b) Reduce the following quadratic terms to their canonical forms.

i) $u_1 = 2x_1^2 - x_2^2 + 3x_1x_2 + x_3^2 - 4x_2x_3$.

UNIT – III

5\. Describe the triangular reduction method and hence solve the equations

$x_1 + x_2 + x_3 = 1$
$4x_1 + 3x_2 - x_3 = 6$
$3x_1 + 5x_2 + 3x_3 = 4$

6\. Show that for any nxn matrix A with distinct eigen values $\lambda_1, \ldots\ldots \lambda_n$

$A = \lambda_1 B_1 + \ldots\ldots + \lambda_n B_n$ where

$I = B_1 + \ldots\ldots B_n$.

$$B_j = \frac{(A-\lambda_1 I)\ldots\ldots(A-\lambda_{j-1} I)\ldots\ldots(A-\lambda_{j+1} I)\ldots(A-\lambda_n I)}{(\lambda_j-\lambda_1)\ldots\ldots(\lambda_j-\lambda_{j-1})(\lambda_j-\lambda_{j+1})\ldots\ldots(\lambda_j-\lambda_n)}$$

and $B_i B_j = 0$ for all $i \neq j$.

UNIT – IV

7. a) Show that every matrix A has a unique Moore-Penrose inverse.

b) Describe with illustration a method for constructing the Moore-Penrose inverse.

8. Define a generalized inverse of a m×n matrix A. If A^- denotes the generalized inverse of A show that

rank $A^- \geq$ rank A.

Describe a method for computation of generalized inverse.

UNIT – V

9. a) Find the iterative method based on Newton-Raphson method for finding $\sqrt{N}$ where N is a real number.

b) Describe the bisection method for finding the root of $f(x) = 0$. Comment on the number of iterations required for converging to a root.

10. a) Describe the Graeffe root squaring method to find the roots of a polynomial with real coefficient.

b) Write short notes on :

i) Efficiency index of an iterative method

ii) Regula-Falsi method.

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