M 25145

Reg. No. : ....

Name : ....

II Semester M.A./M.Sc./M.Com. Degree (Reg./Sup./Imp.) Examination, March 2014
STATISTICS
Paper – 2.4 : Inference – 1

Time : 3 Hours   Max. Marks : 70

Instructions : *1) Answer* ***any 5*** *questions without omitting* ***any*** *Unit.*
2) ***All*** *questions carry* ***equal*** *marks.*

UNIT – I

I. a) Explain unbiasedness and consistency of estimators. If t is consistent estimator of θ then show that t^2 is a consistent estimator of θ^2.

b) Define sufficiency and completeness. Give example of a statistic which is

i) Complete, but not sufficient

ii) Sufficient but not complete. Justify your claim. **(8+6)**

II. a) Illustrate the importance of the property 'efficiency' of an estimator with example.

b) State and prove factorization theorem. **(6+8)**

UNIT – II

III. a) Define MLE and state the regularity condition under which MLE based on a random sample of size n is

i) consistent and

ii) asymptotically normally distributed.

b) Obtain the MLE of θ based on a random sample of size 'n', from the population with p.d.f.

i) $f(x, \theta) = \frac{1}{2} e^{-|x-\theta|}, -\infty < x < \infty$

ii) $f(x, \theta) = \theta (1-x)^{\theta-1}; 0 \le x \le 1, \theta > 1$. **(6+8)**

P.T.O.

IV. a) Estimate the parameters of Gamma distribution by the method of moments.

b) Explain the method of modified minimum chi-square. **(8+6)**

UNIT – III

V. a) State and establish a necessary and sufficient condition for unbiased estimates to be UMVUE.

b) A random sample of size 'n' is taken from $N(0, \sigma^2)$. Find MVUE of σ^2. **(8+6)**

VI. a) State and prove Lehmann-Scheffe theorem.

b) Define MVB estimator. Derive the general form of distributions admitting MVB estimators. **(8+6)**

UNIT – IV

VII. a) Let $X \sim B(n, P)$. The prior distribution of P is $\pi(P) = 1$ for $0 < P < 1$. Find the Bayes estimator of P under squared error loss and also obtain the Bayes risk.

b) Define minimax estimator. Illustrate it with an example. **(7+7)**

VIII.a) Define Bayes estimator. Show that under squared error loss Bayes estimator is the mean of the posterior distribution.

b) Let $X_1, X_2, \ldots X_n$ be i.i.d with p.d.f. $f(x) = \frac{1}{\theta} \exp(-x/\theta)$, $x > 0$. Find the Pitman estimator of θ^k. **(7+7)**

UNIT – V

IX. a) For a sample of size one from the population $f_\theta(x) = \frac{2}{\theta^2}(\theta - x)$, $0 < x < \theta$, find a $(1 - \alpha)$ level confidence interval for θ.

b) Explain the concept of shortest confidence interval. Obtain the shortest confidence interval for θ from $U(0, \theta)$ in terms of the order statistic $X_{(n)}$. **(7+7)**

X. a) Distinguish between UMA and UMAU confidence intervals. If $X_1, X_2, \ldots X_n$ is a random sample from $N(\mu, \sigma^2)$, obtain a UMA confidence interval for σ^2 when μ is unknown.

b) Explain a large sample method of constructing confidence intervals with a suitable illustration. **(8+6)**
