M 23121

Reg. No. : ....

Name : ....

II Semester M.A./M.Sc./M.Com. Degree (Reg./Sup./Imp.) Examination, March 2013
STATISTICS
Paper – 2.4 : Inference – I

Time: 3 Hours Max. Marks : 70

Instructions : *1) Answer **any 5** questions without omitting **any** Unit.*
*2) **All** questions carry **equal** marks.*

UNIT – I

I. a) Define minimal sufficiency and completeness. Show that a complete sufficient statistic is minimal.

b) Let X_1, X_2 be a random sample of size 2 from a Poisson distribution with mean λ. Show that

i) $X_1 + X_2$ is sufficient

ii) $X_1 + 2X_2$ is not sufficient. **(8+6)**

II. a) State and prove factorization theorem.

b) Prove or disprove the following :

i) Unbiased estimators are always consistent.

ii) Consistent estimators are always unbiased. **(8+6)**

UNIT – II

III. a) Discuss the maximum likelihood estimation method and mention its properties.

b) Let X_1, X_2, ..., X_n be a random sample of size n drawn from the Uniform distribution over $(-\theta, \theta)$. Estimate θ by the method of moments and by maximum likelihood method. Compare between these two. **(6+8)**

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IV. a) Find the method of moment estimators of the parameters of Gamma (α, β).

b) Show that under certain conditions, the method of minimum chi-square and maximum likelihood methods give equally efficient estimators. **(8+6)**

UNIT – III

V. a) State and prove Bhattacharya lower bound for the variance of any unbiased estimator of a parameter.

b) State and prove Lehmann-Scheffe theorem. **(8+6)**

VI. a) State Cramer-Rao inequality. Find the Cramer-Rao lower bound for the variance of unbiased estimators of θ, of

$f(x, \theta) = \theta x^{\theta - 1}, 0 < x < 1; \ \theta > 0$.

b) Define minimum variance bound estimator. Derive the general form of distributions admitting minimum variance bound estimators. **(6+8)**

UNIT – IV

VII. a) Obtain the Baye's estimator of P in B(n, P), $0 < p < 1$, if the prior distribution of P is Beta (a, b).

b) What do you mean by minimax estimators ? Illustrate it with an example. **(7+7)**

VIII. a) Explain :

i) Loss function

ii) Risk function

iii) Prior Distribution

iv) Posterior distribution.

b) Let $X_1, X_2, ..., X_n$ be a random sample of size n drawn from $N(\mu, 1)$. The prior distribution of μ is N(0, 1). Find the Baye's estimator of μ under squared error loss function. **(8+6)**

UNIT – V

IX. a) Explain the method of constructing confidence intervals. How is it related with hypothesis testing ?

b) Obtain $100(1-\alpha)\%$ confidence interval for $\mu_1 - \mu_2$, if random sample of sizes n_1 and n_2 are drawn from two independent normal populations $N(\mu_1, \sigma_1^2)$ and $N(\mu_2, \sigma_2^2)$, when

i) σ_1, σ_2 are known

ii) σ_1, σ_2 are unknown. **(6+8)**

X. a) Define UMA confidence bounds. Obtain the UMA upper bound for the mean of a Normal distribution with known variance.

b) Explain the method of constructing large sample confidence intervals. Obtain $100(1-\alpha)\%$ confidence limits for the parameter λ of Poisson distribution. **(7+7)**
